

## Structure of Mathematics Courses for the UG Program

Sem	Name of the courses to be offered by the Department of mathematics				
1	Analysis and Linear Algebra				
2	Analysis and Linear Algebra				
3	Probability and Statistics				
4	Multivariable Calculus and Complex Variable	Elementary Algebra and Number Theory	Elective	Elective	
5	Algebra I	Linear Algebra	Analysis I	Topology I	
6	Analysis II	Complex Analysis	ODE	Elective	
7	Algebraic Topology I	Functional Analysis	PDE	Elective	Elective
8	Project				

Electives include:

Algebra II, Galois Theory, Probability, Harmonic Analysis, Lie Groups, Commutative Algebra, Numerical Analysis, Number Theory.

### Detailed Syllabus:

#### Analysis and Linear Algebra I:

One-variable calculus: Real and Complex numbers; Convergence of sequences and series; Continuity, intermediate value theorem, existence of maxima and minima; Differentiation, mean value theorem, Taylor series; Integration, fundamental theorem of Calculus, improper integrals.

Linear Algebra: Vector spaces (over real and complex numbers), basis and dimension; Linear Transformations and matrices; Determinants.

References:

*Apostol, Calculus, Volume I, 2nd. Edition, Wiley, India, 2007.*

*G. Strang, Linear Algebra And Its Applications, 4th Edition, Brooks/Cole, 2006*

#### Analysis and Linear Algebra:

Linear Algebra continued: Inner products and Orthogonality; Eigenvalues and Eigenvectors; Diagonalisation of Symmetric matrices.

Multivariable calculus: Functions on  $\mathbb{R}^n$ ; Partial and Total derivatives; Chain rule; Maxima, minima and saddles; Lagrange multipliers; Integration in  $\mathbb{R}^n$ , change of variables, Fubini's theorem; Gradient, Divergence and Curl; Line and Surface integrals in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ ; Stokes, Green's and Divergence theorems.

Introduction to Ordinary Differential Equations; Linear ODEs and Canonical forms for linear transformations.

References:

*T. M. Apostol, Calculus, Volume II, 2nd. Edition, Wiley Wiley India, 2007.*

*G. Strang, Linear Algebra And Its Applications, 4th Edition, Brooks/Cole, 2006*

*M. Artin, Algebra, Prentice Hall of India, 1994.*

*M. Hirsch, S. Smale, R. L. Devaney, Differential Equations, Dynamical Systems, and an Introduction to Chaos, 2nd Edition, Academic Press, 2004.*

Probability and Statistics:

Basic notions of probability, conditional probability and independence, Bayes' theorem, random variables and distributions, expectation and variance, conditional expectation, moment generating functions, limit theorems.

Samples and sampling distributions, estimations of parameters, testing of hypotheses, regression, correlation and analysis of variance.

Reference:

*P. L. Meyer, Introductory Probability and Statistical Applications, Addison-Wesley, 1970*

*R. V. Hogg and E. A. Tanis, Probability and Statistical Inference, 2004, Pearson Education Inc., Delhi, Sixth Edition.*

*R. V. Hogg and J. Ledolter, Engineering Statistics, 1987, Macmillan Publishing Company, New York.*

*S. Ross, A First Course in Probability, 2004, Pearson Education Inc., Delhi, Sixth Edition.*

Multivariable Calculus and Complex Variable:

Review of total derivatives, inverse and implicit function theorem; Complex linearity and the Cauchy-Riemann equations; Complex functions, conformal mappings.

Multilinear forms on vector spaces; Tensor products and symmetric products; Alternating forms and the exterior algebra of a vector space.

Differential forms, integration on chains, Stokes theorem for differential forms; Complex integration and Cauchy's theorem; Power series and Laurent series representations; Residue theorem and Contour integration; Harmonic functions.

References:

*T. M. Apostol, Calculus, Volume II, 2nd. Edition, Wiley India, 2007.*

*T. W. Gamelin, Complex Analysis, Springer Undergraduate Texts in Mathematics, Springer International Edition, 2006*

Elementary Algebra and Number Theory:

Divisibility and Euclid's algorithm, Fundamental theorem of Arithmetic, Congruences, Fermat's little theorem and Euler's theorem, the ring of integers modulo  $n$ , factorisation of polynomials, Elementary symmetric functions,

Eisenstein's irreducibility criteria, Formal power series, arithmetic functions, Prime residue class groups, quadratic reciprocity.

Basic concepts of rings, Fields and groups. Applications to number theory.

References:

*D. M. Burton, Elementary number theory, McGraw Hill.*

*I. Niven, H. S. Zuckerman and H. L. Montgomery, An Introduction To The Theory Of Numbers, 5th Edition, Wiley Student Editions*

*G. Fraleigh, A First Course in Abstract Algebra, 7th Edition, Pearson.*

## Algebra I:

Groups: Review of Groups, Subgroups, Homomorphisms, Normal subgroups, Quotient groups, Isomorphism theorems. Group actions and its applications, Sylow theorems. Structure of finitely generated abelian groups, Free groups

Rings: Review of rings, Homomorphisms, Ideals and isomorphism theorems. Prime ideals and maximal ideals. Chinese remainder theorem. Euclidean domains, Principal ideal domains, Unique factorization domains. Factorization in polynomial rings.

Modules: Modules, Homomorphisms and exact sequences. Free modules. Hom and tensor products. Structure theorem for modules over PIDs.

References:

*Lang, S., Algebra, revised third edition. Springer-Verlag, 2002 (Indian Edition Available).*

*Artin, M., Algebra, Prentice-Hall of India, 1994.*

*Dummit, D. S. and Foote, R. M., Abstract Algebra, John Wiley & Sons, 2001.*

*Hungerford, T. W., Algebra, Springer (India), 2004*

*Herstein, I. N., Topics in Algebra, John Wiley & Sons, 1995.*

## Linear Algebra:

Vector spaces: Basis and dimension, Direct sums.

Determinants: Theory of determinants, Cramer's rule.

Linear transformations: Rank-nullity theorem, Algebra of linear transformations, Dual spaces. Linear operators, Eigenvalues and eigenvectors, Characteristic polynomial, Cayley-Hamilton theorem, Minimal polynomial, Algebraic and geometric multiplicities, Diagonalization, Jordan canonical Form.

Symmetry: Group of motions of the plane, Discrete groups of motion, Finite groups of  $S_0(3)$ .

Bilinear forms: Symmetric, skew symmetric and Hermitian forms, Sylvester's law of inertia, Spectral theorem for the Hermitian and normal operators on finite dimensional vector spaces.

Linear groups: Classical linear groups,  $SU_2$  and  $SL_2(\mathbb{R})$ .

References:

Artin, M., Algebra, Prentice-Hall of India, 1994.

Herstein, I. N., Topics in Algebra, Vikas Publications, 1972.

Strang, G., Linear Algebra and its Applications, Third Edition, Saunders, 1988.

Halmos, P., Finite dimensional vector spaces, Springer-Verlag (UTM), 1987.

### Analysis I:

Review of Real and Complex numbers systems, Topology of  $\mathbb{R}$ , Continuity and differentiability, Mean value theorem, Intermediate value theorem. The Riemann-Stieltjes integral. Introduction to functions of several variables, differentiability, directional and total derivatives. Sequences and series of functions, uniform convergence, the Weierstrass approximation theorem.

#### References:

*Rudin, W., Principles of Mathematical Analysis, McGraw-Hill, 1986.*

*Royden, H. L., Real Analysis, Macmillan, 1988.*

### Topology I:

Open and closed sets, continuous functions, the metric topology, the product topology, the ordered topology, the quotient topology. Connectedness and path connectedness, local path connectedness. Compactness. Countability axioms. Separation axioms. Complete metric spaces, the Baire category theorem. Urysohn's embedding theorem. Function. Topological groups, orbit spaces.

#### References

*Armstrong, M. A., Basic Topology, Springer (India), 2004.*

*Janich, K., Topology, Springer-Verlag (UTM), 1984.*

*Munkres, K. R., Topology, Pearson Education, 2005.*

*Simmons, G. F., Topology and Modern Analysis, McGraw-Hill, 1963.*

### Analysis II:

Construction of the Lebesgue measure, measurable functions, limit theorems. Lebesgue integration. Different notions of convergence and convergence theorems. Product measures and the Radon-Nikodym theorem, change of variables, complex measures.

#### References:

*De Berra, G., Introduction to Measure Theory, Von Nostrand Reinhold, 1974.*

*Hewitt, E. and Stromberg, K., Real and Abstract Analysis, Springer, 1969*

*Royden, H. L., Real Analysis, Macmillan, 1988.*

*Rudin, W., Real and Complex Analysis, McGraw-Hill, 1986.*

### Complex Analysis:

Complex numbers, complex-analytic functions, Cauchy's integral formula, power series, Liouville's theorem. The maximum-modulus theorem. Isolated singularities, residue theorem, the

Argument Principle, real integrals via contour integration. Mobius transformations, conformal mappings. The Schwarz lemma, automorphisms of the disc. Normal families and Montel's theorem. The Riemann mapping theorem.

References:

*Ahlfors, L. V., Complex Analysis, McGraw-Hill, 1979.*

*Conway, J. B., Functions of One Complex Variable, Springer-Verlag, 1978.*

*Gamelin, T.W., Complex Analysis, UTM, Springer, 2001.*

ODE:

Basics concepts: Phase space, existence and uniqueness theorems, dependence on initial conditions, flows.

Linear system: The fundamental matrix, stability of equilibrium points. Sturm-Liouville theory .

Nonlinear system and their stability: The Poincaré-Bendixon theorem, perturbed linear systems, Lyapunov methods.

References:

*Coddington, E. A. and Levinson, N., Theory of Ordinary Differential Equations, Tata McGraw-Hill 1972.*

*Birkhoff, G. and Rota, G.-C., Ordinary Differential Equations, Wiley, 1989.*

*Hartman, P., Ordinary Differential Equations, Birkhäuser, 1982.*

Algebraic Topology I:

The fundamental group: Homotopy of maps, multiplication of paths, the fundamental group, induced homomorphisms, the fundamental group of the circle, covering spaces, lifting theorems, the universal covering space, Seifert-Van Kampen theorem, applications.

Simplicial Complexes, Simplicial and Singular homology - Definitions, Properties and Applications.

References:

Armstrong, M. A., Basic Topology, Springer (India), 2004.

Hatcher, A., Algebraic Topology, Cambridge Univ. Press, 2002.

Kosniowski, C., A First Course in Algebraic Topology, Cambridge Univ. Press, 1980.

Munkres, K. R., Topology, Pearson Education, 2005.

Greenberg, M. J., Lectures on Algebraic Topology. W. A. Benjamin Inc., London, 1973.

Munkres, J. R., Elements of Algebraic Topology, Addison-Wesley, 1984.

*Spanier, E. H., Algebraic Topology, tata McGraw-Hill, 1966*

Functional Analysis:

Basic topological concepts, Metric spaces, Normed linear spaces, Banach spaces, Bounded linear functionals and dual spaces, Hahn-Banach theorem. Bounded linear operators, open-mapping theorem, closed graph theorem. The Banach-Steinhaus theorem. Hilbert spaces, Riesz representation theorem, orthogonal complements, bounded operators on a Hilbert space. The spectral theorem for compact, self-adjoint operators.

References:

*Goffman, C. and Pedrick, G., First course in Functional Analysis, Prentice-Hall of India, 1995.*

*Conway, J. B., A course in Functional Analysis, Springer, 1990.*

*Taylor, A. E., Introduction to Functional Analysis, Wiley International Edition, 1958.*

PDE:

First order partial differential equation and Hamilton-Jacobi equations; Cauchy problem and classification of second order equations, Holmgren's uniqueness theorem; Laplace equation; Diffusion equation; Wave equation; Some methods of solutions, Variable separable method.

References:

*Garabedian, P. R., Partial Differential Equations, John Wiley and Sons, 1964.*

*Prasad. P. and Ravindran, R., Partial Differential Equations, Wiley Eastern, 1985.*

*Renardy, M. and Rogers, R. C., An Introduction to Partial Differential Equations, Springer-Verlag, 1992.*

*Fritz John, Partial differential Equations, Springer (International Students Edition), 1971.*