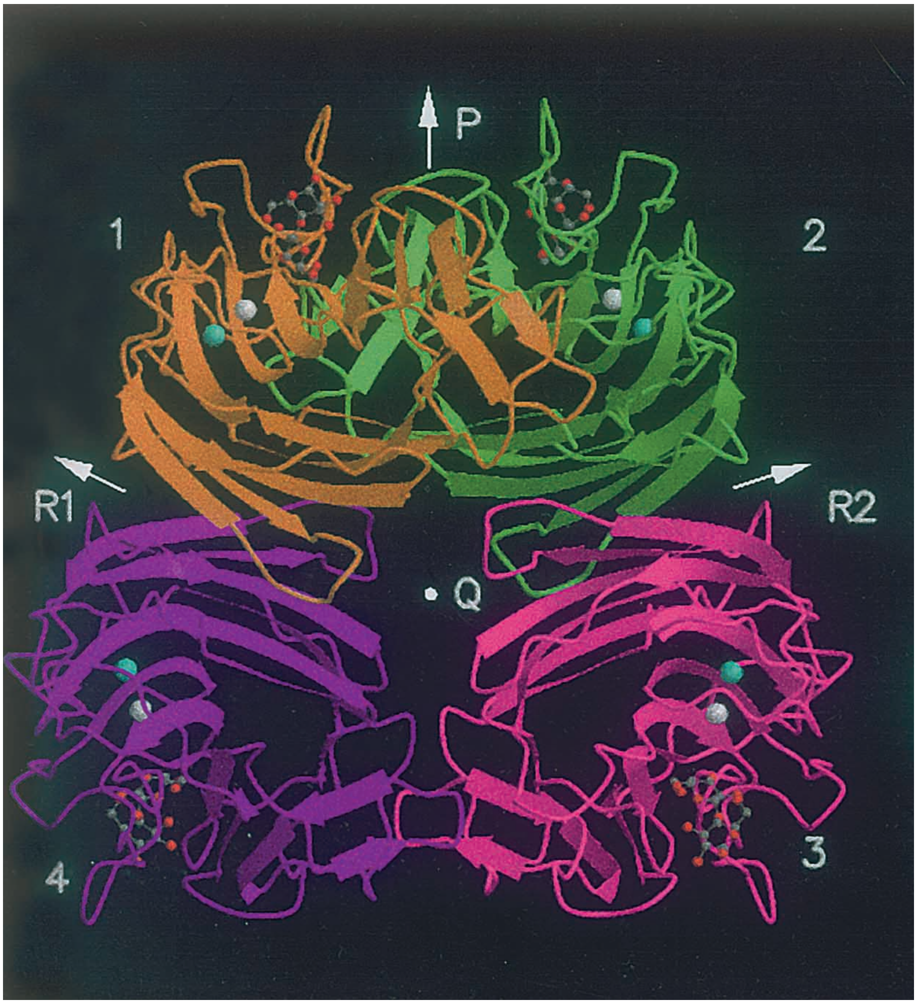
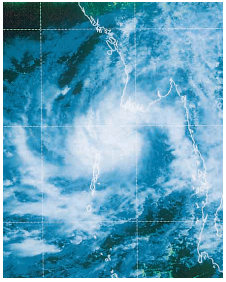


IN SEARCH OF EXCELLENCE

Excellence is a journey and not a destination. In science it implies perpetual efforts to advance the frontiers of knowledge. This often leads to progressively increasing specialization and emergence of newer disciplines. A brief summary of salient contributions of Indian scientists in various disciplines is introduced in this section.



CHAPTER VII

MATHEMATICAL SCIENCES

The modern period of mathematics research in India started with Srinivasa Ramanujan whose work on analytic number theory and modular forms is highly relevant even today. In the pre-Independence period mathematicians like S.S. Pillai, Vaidyanathaswamy, Ananda Rau and others contributed a lot.

Particular mention should be made of universities in Allahabad, Varanasi, Kolkata, Chennai and Waltair and later at Chandigarh, Hyderabad, Bangalore and Delhi (JNU). The Department of Atomic Energy came in a big way to boost mathematical research by starting and nurturing the Tata Institute of Fundamental Research (TIFR), which, under the leadership of Chandrasekharan, blossomed into a great school of learning of international standard. The Indian Statistical Institute, started by P.C. Mahalanobis, made its mark in an international scene and continues to flourish. Applied mathematics community owes a great deal to the services of three giants — N.R. Sen, B.R. Seth and P.L. Khastgir. Some of the areas in which significant contributions have been made are briefly described here.

ALGEBRA

One might say that the work on modern algebra in India started with the beautiful piece of work in 1958 on the proof of Serre's conjecture for n=2. A particular case of the conjecture is to imply that a unimodular vector with polynomial entries in n variables can be completed to a matrix of determinant one. Another important school from India was started in Panjab University whose work centres around Zassanhaus conjecture on groupings.

ALGEBRAIC GEOMETRY

The study of algebraic geometry began with a seminal paper in 1964 on vector bundles. With further study on vector bundles that led to the moduli of parabolic bundles, principle bundles, algebraic differential equations (and more recently the relationship with string theory and physics), TIFR has become a leading school in algebraic geometry. Of the later generation, two pieces of work need special mention: the work on characterization of affine plane purely topologically as a smooth affine surface, simply connected at infinity and the work on Kodaira vanishing. There is also some work giving purely algebraic geometry description of the topologically invariants of algebraic varieties. In particular this can be used to study the Galois Module Structure of these invariants.

LIE THEORY

The inspiration of a work in Lie theory in India came from the monumental work on infinite dimensional representation theory by Harish Chandra, who has, in some sense, brought the subject from the periphery of mathematics to centre stage. In India, the initial study was on the discrete subgroups of Lie groups from number theoretic angle. The subject received an impetus after an international conference in 1960 in TIFR, where the leading lights on the subject, including A. Selberg participated. Then work on rigidity questions was initiated. The question is whether the lattices in arithmetic groups can have interesting deformations except for the well-known classical cases. Many important cases in this question were settled.

Differential Equation

After the study of *L*-functions were found to be useful in number theory and arithmetic geometry, it became natural to study the *L*-functions arising out of the eigenvalues of discrete spectrum of the differential equations. Minakshisundaram's result on the corresponding result for the differential equation leading to the Epstein Zeta function and his paper with A. Pleijel on the same for the connected compact Riemanian manifold are works of great importance. The idea of the paper (namely using the heat equation) lead to further improvement in the hands of Patodi. The results on regularity of weak solution is an important piece of work. In the later 1970s a good school on non-linear partial differential equations that was set up as a joint venture between TIFR and IISc, has come up very well and an impressive lists of results to its credit.

For differential equations in applied mathematics, the result of P.L. Bhatnagar, BGK model (by Bhatnagar, Gross, Krook) in collision process in gas and an explanation of Ramdas Paradox (that the temperature minimum happens about 30 cm above the surface) will stand out as good mathematical models. Further significant contributions have been made to the area of group theoretic methods for the exact solutions of nonliner partial differential equations of physical and engineering systems.

Ergodic Theory

Earliest important contribution to the Ergodic theory in India came from the Indian Statistical Institute. Around 1970, there was work on spectra of unitary operators associated to non-singular transformation of flows and their twisted version, involving a cocycle.

Two results in the subjects from 1980s and 1990s are quoted. If G is lattice in SL (2,R) and {u_t} a unipotent one parameter subgroup of G, then all non-periodic orbits of {u_t} on G_1 are uniformly distributed. If Q is non-generate in definite quadratic form in n=variables, which is not a multiple of rational form, then the number of lattice points x_ with $a < |Q(x)| < b$, $||x|| < r$, is at least comparable to the volume of the corresponding region.

Number Theory

The tradition on number theory started with Ramanujan. His work on the cusp form for the full modular group was a breakthrough in the study of modular form. His conjectures on the coefficient of this cusp form (called Ramanujan's tau function) and the connection of these conjectures with conjectures of A. Weil in algebraic geometry opened new research areas in mathematics. Ramanujan's work (with Hardy) on an asymptotic formula for the partition of *n*, led a new approach (in the hands of Hardy-Littlewood) to attack such problems called circle method. This idea was further refined and S.S. Pillai settled Waring's Conjecture for the 6th power by this method. Later the only remaining case namely 4th powers was settled in mid-1980s. After Independence, the major work in number theory was in analytic number theory, by the school in TIFR and in geometry of numbers by the school in Panjab University. The work on elliptic units and the construction of ray class fields over imaginary quadratic fields of elliptic units are some of the important achievements of Indian number theory school. Pioneering work in Baker's Theory of linear forms in logarithms and work on geometry of numbers and in particular the Minkowski's theorem for n = 5 are worth mentioning.

Probability Theory

Some of the landmarks in research in probability theory at the Indian Statistical Institute are the following:

- A comprehensive study of the topology of weak convergence in the space of probability measures on topological spaces, particularly, metric spaces. This includes central limit theorems in locally compact abelian groups and Milhert spaces, arithmetic of probability distributions under convolution in topological groups, Levy-khichini representations for characteristic functions of probability distributions on group and vector spaces.
- Characterization problems of mathematical statistics with emphasis on the derivation of probability laws under natural constraints on statistics evaluated from independent observations.
- Development of quantum stochastic calculus based on a quantum version of Ito's formula for non-commutative stochastic processes in Fock spaces. This includes the study of quantum stochastic integrals and differential equations leading to the construction of operator Markov processes describing the evolution of irreversible quantum processes.
- Martingale methods in the study of diffusion processes in infinite dimensional spaces.
- Stochastic processes in financial mathematics.

Combinatorics

Though the work in combinatorics had been initiated in India purely through the efforts of R.C.Bose at the Indian Statistical Institute in late thirties, it reached its peak in late fifties at the University of North Carolina, USA, where he was joined by his former student S.S.Shrikhande. They provided the first counter-example to the celebrated conjecture of Euler (1782) and jointly with Parker further improved it. The last result is regarded a classic.

In the absence of these giants there was practically no research activity in this area in India. However, with the return of Shrikhande to India in 1960 activities in the area flourished and many notable results in the areas of embedding of residual designs in symmetric designs, A-design conjecture and t-designs and codes were reported.

Theory of Relativity

In a strict sense the subject falls well within the purview of physics but due to the overwhelming response by workers with strong foundation in applied mathematics the activity could blossom in some of the departments of mathematics of certain universities/institutes. Groups in BHU, Gujarat University, Ahmedabad, Calcutta University, and IIT, Kharagpur, have contributed generously to the area of exact solutions of Einstein equations of general relativity, unified field theory and others. However, one exact solution which has come to be known as Vaidya metric and seems to have wide application in high-energy astrophysics deserves a special mention.

Numerical Analysis

The work in this area commenced with an attempt to solve non-linear partial differential equations governing many a physical and engineering system with special reference to the study of Navier-Stabes equations and cross-viscous forces in non-Newtonian fluids. The work on N-S equation has turned out to be a basic paper in the sense that it reappeared in the volume, *Selected Papers on Numerical Solution of Equations of Fluid Dynamics, Applied Mathematics*, through the Physical Society of Japan. The work on non-Newtonian fluid has found a place in the most prestigious volume on *Principles of Classical Mechanics & Field Theory* by Truesdell and Toupin. The other works which deserve mention are the development of extremal point collocation method and stiffy stable method.

Applied Mathematics

Till 1950, except for a group of research enthusiasts working under the guidance of N.R.Sen at Calcutta University there was practically no output in applied mathematics. However, with directives from the centre to emphasize on research in basic and applied sciences and liberal central fundings through central and state sponsored laboratories, the activity did receive an impetus. The department of mathematics at IIT, Kharagpur, established at the very inception of the institute of national importance in 1951, under the dynamic leadership of B.R.Seth took the lead role in developing a group of excellence in certain areas of mathematical sciences. In fact, the research carried out there in various disciplines of applied mathematics such as elasticity-plasticity, non-linear mechanics, rheological fluid mechanics, hydroelasticity, thermoelasticity, numerical analysis, theory of relativity, cosmology, magneto hydrodynamics and high-temperature gasdynamics turned out to be a trend setting one for other IITs, RECs, other Technical Institutes and Universities that were in the formative stages. B.R. Seth's own researches on the study of Saint-Venamt's problem and transition theory to unify elastic-plastic behaviour of materials earned him the prestigious Euler's bronze medal of the Soviet Academy of Sciences in 1957. The other areas in which applied mathematicians contributed generously are biomechanics, CFD, chaotic dynamics, theory of turbulence, bifurcation analysis, porous media, magnetics fluids and mathematical physiology.