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Multiple criteria decision making: Assigning teachers – an example

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Assignment problems when more than one criterion are to be used in assigning personnel have drawn little attention in the literature, even though there are important practical problems coming under this category. This paper addresses a problem of assigning personnel giving due consideration to preferences and other relevant criteria. An algorithm to satisfactorily solve a problem encountered in the educational sector in Sri Lanka is presented.

SOME areas in a country may be considered as congenial while others as difficult because of lack of certain basic facilities. People prefer to serve in congenial areas due to various reasons. It is assumed that in congenial areas health, educational, transportation, housing and entertainment facilities are better. Also sometimes certain areas become difficult due to adverse climatic conditions alone. Normally congenial areas are assumed to have a higher population density than in uncongenial or difficult areas. This is especially true in developing countries where resources are limited in order to develop all the regions with equal infrastructure facilities.

As a result, more and more educated people originate from congenial areas because the educational facilities in these areas are better compared to those in uncongenial areas. It is also assumed that the majority of the professionals prefer to work in congenial areas. Here we take the teaching profession as an example. Let us assume that a certain number of candidates have been selected to be employed as probationary teachers by the relevant authorities. They will have to be posted to schools in different areas of a country. It is also assumed that, a country is divided into several educational regions or districts. It appears that there is a natural tendency for a majority of them to prefer appointments in places closer to their home towns or in congenial areas. Sometimes, even the candidates coming from difficult areas seem to prefer to work in congenial areas due to the availability of certain facilities which enable them for advancement in their careers. We assume that the prospective candidates are given the opportunity to indicate their preferences for districts in their application forms.

However, it has been found that it is almost impossible to heed to these requests, as too many candidates are opting to serve in congenial areas. Usually, the number of qualified candidates hailing from congenial areas is more than the number of vacancies existing in these areas. On the other hand, very few qualified candidates originate from uncongenial areas, far short of those required in these areas. Therefore, assigning teachers to schools satisfying the preferences has become a difficult problem in Sri Lanka. One way to solve this problem is to impose the condition that every new recruit should serve a minimum stipulated period in uncongenial areas. However, this policy could lead to other complications. In fact, in Sri Lanka this condition has been imposed sometime ago when recruiting new teachers but the method was not successful. It was found that most of the teachers sent to difficult areas against their will, lacked motivation and this affected the quality of education of students. Besides, these teachers used to take excessive amounts of leave, thus making students to miss their lessons often. Therefore, it is necessary to device a better method for assigning teachers to schools.

A somewhat similar assignment problem has been addressed in the literature[1]. It is an assignment problem with seniority and job priority constraints. Here, it is assumed that candidates belong to given seniority classes and jobs have given priority levels. Also, it is considered that the candidates are qualified for some but usually not for all of the jobs. As a result, our problem differs from this in several ways; for example, we assume that the candidates are qualified for all vacancies and we consider conflicting goals as objectives.

Multiple criteria

In this paper, we consider a problem which makes use of several criteria at different levels for assigning teachers to various districts. It is expected that the proposed method of assignment will be fair for all the teachers concerned while trying to satisfy as far as possible, their preferences. The candidates to be appointed as teachers in a particular category are required to apply for vacancies in different districts according to their preferences. The number of preferences allowed will be decided by the relevant authorities. It is assumed that the recruitment is centralized and the requirements of districts are pooled together to assign the teachers.

In assigning teachers, the criteria we consider are the teachers' preferences, the marks allocated to them based on their qualifications, experience, etc. and the distance to a central location of the districts from their home-towns. As the criteria considered here may be conflicting, it is not possible to arrive at an overall optimal solution to this problem. Therefore, common linear programming approaches which are designed to solve single objective optimization problems could not be used to solve this problem. Hence, we consider a goal programming approach which will give a compromise solution.

Method of approach

In the proposed model, preferences of the candidate are given the highest priority, as it is assumed that the candidate has considered various personal issues of interest to him in deciding these. Marks of the candidates are considered the second priority and these will be used only when the number of applicants for a particular district is more than the existing number of vacancies.

For each candidate, the distance of the district from candidate's home-town is assigned the lowest priority; it is considered only to assign the remaining candidates for whom districts could not be offered based on earlier criteria. It is also possible that the candidate has considered the distance criterion already as one of the issues when he makes his preferences. But, here the distance is considered by the authorities in order to make the maximum possible concession to a candidate who could not be assigned a district based on the earlier criteria.

The proposed model tries to assign every teacher to the district of his/her first choice. When this is not possible due to restrictions in the number of vacancies, a candidate will be assigned to the district of his/her next choice and so on. If the number of preferences allowed is less than the number of districts in which vacancies are available, there will be some vacancies for which candidates could not be assigned based on the preferences alone. This situation occurs when the number of candidates with a particular preference level for a certain district becomes greater than the number of existing vacancies in that district. For these vacancies, marks scored by the candidates will be used for assigning them. The candidates with a higher mark on this scale will be given priority in filling the remaining positions with respect to a set of candidates with a given preference level. This means that, the consideration of marks will be embedded within preference levels, not as a separate goal.

Even after these assignments, a set of vacancies may still exist which could not be filled using the above two criteria. These vacancies will be filled by considering the remaining candidates who will be assigned districts as close as possible to their home-towns.

The order in which districts are considered may affect the assignments. However, here we assume that the order in which the authorities have listed the districts is considered as the relevant order. Therefore, it is possible that the proposed method may not give an optimal solution to the problem, but it will give rise to a reasonably good solution that can be applied in real situations.

Mathematical model

Let m be the number of candidates; n, the number of districts or regions; q, the number of preference levels allowed; p_i, the marks awarded to candidate i (= mark[i]) based on qualifications, experience, etc.; d_{ij}, the distance of the district j from the home-town of candidate i (distance from a central location of the district may be taken here).

c_{ij}, the preference of candidate i to district j. If q preferences are allowed, c_{ij} may take values $k = 1, 2, \ldots, q$ (k is the preference level, $k = 1$ indicates the highest preference; pref[k] = number of candidates with preference k).

v_j, number of vacancies in district j (= vacant[j]) where Σv_j, is the total number of vacancies;

$$x_{ij} = \begin{cases} 1 & \text{if teacher } i \text{ is assigned to district } j \\ 0 & \text{otherwise} \end{cases}.$$

We consider a goal programming formulation consisting of two main objectives: (i) Maximize the number of candidates who have been awarded their first preference. (ii) Minimize the distance involved when selecting candidates based on distance consideration.

In order to convert into a goal program, we must first establish aspiration levels for each objective. These aspiration levels are values that we might hope to achieve in the final solution[2]. The solution procedure consists of solving one goal problem at a time, starting with the highest priority goal and terminating with the lowest priority goal[3]. The process is carried out in such a way that the solution obtained from a lower priority goal does not degrade any of the solutions already secured for the higher priority goals. In solving the goal programming problem, what we try to achieve is to obtain a solution which leads to the minimum deviation from the specified goals.

Let us consider the highest priority goal of maximizing the number of candidates who have been awarded their first preference. It is obvious that, we may not be able to offer all candidates, the districts of their first choice. Therefore, here we take our goal as at least t% of the total number of vacancies should be filled with the candidates with their first preference. This gives us the inequality,

$$\sum_i \sum_j c_{ij} x_{ij} \geq t \sum_j v_j \quad \text{for candidates assigned with } c_{ij} = 1.$$

For the second goal, we assume that the sum of the relevant distances for candidates selected by distance consideration should be less than the sum of the average distance for each candidate for different districts, i.e.

$$\sum_i \sum_j d_{ij} x_{ij} \leq \sum_i a_i$$

for candidates assigned by considering distances.

Here, a_i is the average distance to districts with vacancies for candidate i.

Candidates will be assigned districts either according to preferences (sometimes considering marks with preferences if necessary) or distances but not both. Each of the above inequalities represents a goal that we should try to satisfy. Since these goals are conflicting, we have to try to reach a compromise solution. For this, we shall first convert each inequality into a flexible goal in which the constraints may be violated, if necessary. These flexible goals can be expressed as follows.

$$\sum_i \sum_j c_{ij} x_{ij} + s_1^+ - s_1^- = t \sum_j v_j$$

for candidates assigned with $c_{ij} = 1$.

$$\sum_i \sum_j d_{ij} x_{ij} + s_2^+ - s_2^- = \sum_i a_i$$

for candidates assigned by considering distance.

$s_1^+, s_1^-, s_2^+, s_2^- \geq 0$ are deviational variables which represent the deviations above and below the right-hand side of constraints. s_1^+, s_2^- represent the amounts by which the respective goals may be violated. Thus, the compromise solution that we are looking for should seek to satisfy the following goals as far as possible:

Goal 1: Minimize the deviation $(G_1 = s_1^+)$ from the desirable number of candidates who would be given appointments according to their first preference.
Goal 2: Minimize the deviation $(G_2 = s_2^-)$ from the sum of the average distance with respect to each candidate for different districts.

Hence, according to the principles of goal programming these goals must be considered simultaneously. Thus the compromise solution that we are looking for should seek to satisfy these goals as far as possible.

Then, the complete mathematical model for the problem may be stated as follows:

Minimize $G_1 = s_1^+$

Minimize $G_2 = s_2^-$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \text{ for } i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = v_j, \text{ for } j = 1, 2, \ldots, n$$

$$\sum_i \sum_j c_{ij} x_{ij} + s_1^+ - s_1^- = t \sum_j v_j,$$

$$\sum_i \sum_j d_{ij} x_{ij} + s_2^+ - s_2^- = \sum_i a_i,$$

$x_{ij} = 0$ or 1.

$s_1^+, s_1^-, s_2^+, s_2^- \geq 0.$

Algorithm

Step (0): (Initialization)

Let $j = 1$ (district number); $k = 1$ (highest preference level).

Step (1):

If vacant[j] = 0, go to step (2).

If vacant[j] > 0, *then*

if the number of candidates with preference k (= pref[k]) not yet assigned is ≤ vacant[j]

then assign all candidates with preference k to district j and update vacant[j];

else assign only candidates which would fill all vacancies in district j according to the decreasing order of mark[i];
let vacant[j] = 0;

Step (2):

Let $j = j + 1$. If $j \leq n$,

then go to step (1),

else let $k = k + 1$. If $k \leq q$, let $j = 1$ and go to step (1).

Step (3): (assigning candidates according to distance from districts)

for each unassigned candidate *do*

among districts with vacant[j] > 0, select a district associated with the shortest possible distance and assign a candidate to it. Update vacant[j].

Description of the algorithm

We divide the set of all districts into two groups A and B. We assume that the group A contains the set of districts still having vacancies, while the group B contains the set of districts in which all vacancies have been filled. Initially, all districts belong to group A, and group B is empty.

The algorithm in the first phase scans through all the districts 1, 2, . . ., n starting from district 1. For each district, the total number of candidates with the first preference for that district is recorded. If this number is less than or equal to the number of vacancies existing in that district, all the candidates are assigned to the district of their first choice. Then, the number of vacancies in the district is updated. However, if the number of candidates with the first preference for a district is greater than the number of vacancies in the district j (vacant[j]), the number of candidates equal to vacant[j] is chosen to fill these positions from those who have scored the highest marks (mark[i]). In order to do this, it is necessary to rank these candidates according to the nonincreasing order of marks scored by them (rank 1 is given to the candidate with the highest marks). Once this is done, the vacancies in that district will be exhausted and it will be moved to group B. This process is continued for all districts.

After this, only the districts in group A will still have vacancies and all possible candidates with the first preference have been assigned. Now the algorithm tries to assign the candidates with the second preference. Here again, the same procedure is followed and the candidates with the second preference are picked from those candidates who have not already been assigned. A district in group A is moved into group B once all the vacancies in that district are filled. The process is continued until all the preferences are exhausted.

Now, the algorithm enters the second phase. The candidates not assigned so far will have to be assigned districts taking into consideration the distance of their home towns from (a central location of) each district. For each candidate, the distances to districts in which vacancies exist, are recorded in the ascending order together with the relevant district number. An attempt is made to fill the vacancies in each district, with candidates associated with the lower distances. The order of the candidates are kept as in the original list. Each unassigned candidate is assigned to a district associated with the least possible distance. However, it may not be possible to assign all these candidates to districts with the lowest distances, as the vacancies are being filled up gradually. A flowchart illustrating the algorithm is given in Figure 1.

Numerical example

Let us now consider the following problem of assigning 24 (= m) candidates to 6 (= n) districts. It is assumed that only 3 preferences (= q) are allowed. Assume that the percentage of vacancies desired to be filled using first preference (= t) is 75%.

District no. (j =)	1	2	3	4	5	6
No. of vacancies (vacant[j] =)	5	7	4	3	6	5

Thus the total number of vacancies (Σv_j) is 30.

Candidate no. (i =)	1	2	3	4	5	6	7	8
Preference order	(2, 1, 3)	(2, 1, 3)	(1, 2, 3)	(3, 2, 4)	(2, 3, 1)	(3, 2, 1)	(2, 1, 3)	(2, 1, 3)
Marks (mark[i])	25	32	47	19	36	41	53	50

Candidate no. (i)	9	10	11	12	13	14	15	16
Preference order	(1, 2, 3)	(4, 3, 5)	(2, 3, 4)	(1, 2, 4)	(2, 3, 1)	(2, 1, 3)	(1, 2, 3)	(2, 1, 3)
Marks (mark[i])	33	31	22	28	29	35	34	37

Candidate no. (i)	17	18	19	20	21	22	23	24
Preference order	(1, 2, 4)	(2, 3, 4)	(1, 2, 3)	(2, 1, 4)	(1, 2, 3)	(2, 1, 5)	(2, 3, 1)	(1, 2, 3)
Marks (mark[i])	30	48	51	39	43	37	24	27

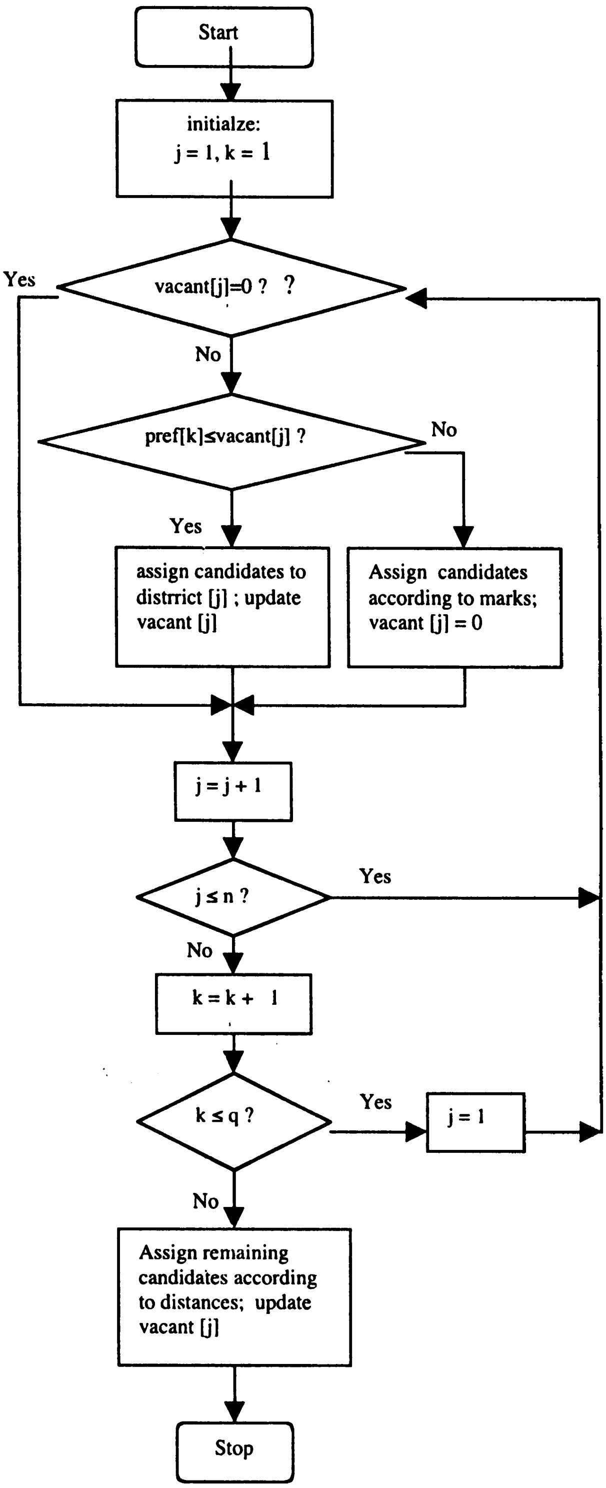

Figure 1. Flowchart for assigning teachers.

From the above data it can be seen that the total number of preferences for each district are as follows:

District no.	Total vacancies	No. of candidates with preference		
		1	2	3
1	5	8	8	3
2	7	13	10	0
3	4	2	6	13
4	3	1	0	6
5	6	0	0	2
6	5	0	0	0

Distance to each district j for each candidate i is also assumed to be known.

After completing the first phase of the algorithm, assignments obtained are given below.

District	Numbers of assigned candidates (Pref. 1)	(Pref. 2)	(Pref. 3)
1	3, 9, 15, 19, 21	–	–
2	5, 7, 8, 16, 18, 20, 22	–	–
3	4, 6	13, 23	–
4	10	–	12, 17
5	–	–	–
6	–	–	–

At this stage, all possible candidates have been assigned districts according to preferences. As we can see, no candidates have been assigned to districts 5 and 6. The candidates not assigned so far correspond to candidate numbers: i = 1, 2, 11, 14, 24. These candidates will have to be assigned districts taking into consideration the distance of each district from their home-towns. The number of remaining vacancies in districts 5 and 6 are 6 and 5, respectively.

Only the distances with respect to the unassigned candidates for the relevant districts are mentioned below to save space:

Candidate no.	Distance to district 5	6	Average distance
1	50	75	62.5
2	70	60	65.0
11	120	145	132.5
14	90	130	110.0
24	75	85	80.0

Underlined numbers indicate the distance with respect to a selected candidate.

Accordingly, in the second phase of the algorithm,

District 5 is assigned to candidates: i = 1, 11, 14, 24.
District 6 is assigned to candidates i = 2.

This finishes the assigning of all candidates.

Output

Candidate no. (i)	1	2	3	4	5	6	7	8
District assigned (j)	5	6	1	3	2	3	2	2
Preference (k)	–	–	1	1	1	1	1	1
Candidate no. (i)	9	10	11	12	13	14	15	16
District assigned (j)	1	4	5	4	3	5	1	2
Preference (k)	1	1	–	3	2	–	1	1
Candidate no. (i)	17	18	19	20	21	22	23	24
District assigned (j)	4	2	1	2	1	2	3	5
Preference (k)	3	1	1	1	1	1	2	–

From the above it can be seen that out of 24 candidates, 15 have been assigned to the districts of their first preference, while 2 candidates each received the districts of their second and third preferences respectively. Only 5 candidates could not be assigned districts according to any of their preferences. They have been assigned districts according to the distances to respective districts. At the end of the assignment it can be seen that two vacancies in district 5 and four vacancies in district 6 remain unfilled.

Algorithm performance

From the above results, the level of achievement of the goals could be computed. For candidates assigned with

$$c_{ij} = 1, \sum_i \sum_j x_{ij} = 15,$$

i.e. the number of candidates for which the first preference has been awarded.

The total number of vacancies = $\sum_j v_j = 24$,

and

$$t\sum_j v_j = \left(\frac{75}{100}\right) 24 = 18.$$

Then, from

$$\sum_i \sum_j x_{ij} + s_1^+ - s_1^- = t\sum_j v_j ,$$

$$s_1^+ = 3 \text{ and } s_1^- = 0. \text{ Min } G_1 = 3.$$

Hence, the deviation from the goal G_1 in the solution is $s_1^+ = 3$.

$$\sum_i a_i = 62.5 + 65.0 + 132.5 + 110.0 + 80 = 450,$$

$$\sum_i \sum_j d_{ij} x_{ij} = 50 + 60 + 120 + 90 + 75 = 395.$$

Then, from

$$\sum_i \sum_j d_{ij} x_{ij} + s_2^+ - s_2^- = \sum_i a_i , s_2^+ = 45 \text{ and } s_2^- = 0.$$

The deviation from the goal G_2 in the solution is $s_2^- = 0$. Min $G_2 = 0$.

Therefore, we can see that the above solution achieves goal G_1 by 83.3% while the goal G_2 is achieved completely (100%). The gap between the goal G_1 and the solution achieved could be further narrowed down by suitably selecting the aspiration levels which may be realistic depending on the application.

In step (1) of the algorithm, if the number of candidates with preference k not yet assigned is > vacant[j], it has to sort the marks obtained by the candidates which could be done by $O(n^2)$ computations. Again, when assigning candidates according to distances from districts similar sorting may have to be undertaken. Hence, in the worst case the algorithm requires $O(n^2)$ computations. Therefore, any large problem could be handled without difficulty.

Conclusion

The problem of assigning prospective candidates in a certain profession to vacancies in different regions or districts in a country was addressed in this paper. The basic criterion for assignment was the preference of candidates for different districts. When it was not possible to assign candidates based on preferences alone, criteria like marks obtained by candidates (from experience, qualification, etc.) and distances of different districts from their hometowns have been taken into account. The algorithm provides a satisfactory solution to a practical problem with respect to teacher assignments in Sri Lanka. It is expected that this algorithm will be useful to solve similar problems arising in other areas too.

Even though the above scheme provides a satisfactory solution to the teacher assignment problem, the motivation of the teachers will have to be raised in order for them to improve the quality of education in under-developed districts. This could be done by providing monetary incentives and further promotional prospects for the teachers serving in the difficult areas.

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