On Einstein's resolution of the twin clock paradox

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Einstein addressed the twin paradox in special relativity in a relatively unknown, unusual and rarely cited paper written in 1918, in the form of a dialogue between a critic and a relativist. Contrary to most textbook versions of the resolution, Einstein admitted that the special relativistic time dilation was symmetric for the twins, and he had to invoke, asymmetrically, the general relativistic gravitational time dilation during the brief periods of acceleration to justify the asymmetrical aging. Notably, Einstein did not use any argument related to simultaneity or Doppler shift in his analysis. I discuss Einstein's resolution and several conceptual issues that arise. It is concluded that Einstein's resolution using gravitational time dilation suffers from logical and physical flaws, and gives incorrect answers in a general setting. The counter examples imply the need to reconsider many issues related to the comparison of transported clocks. The failure of the accepted views and resolutions is traced to the fact that the special relativity principle formulated originally for physics in empty space is not valid in the matter-filled universe.

Keywords: Cosmic relativity, general relativity, gravitational time dilation, special relativity, twin paradox.

The twin clock paradox

EINSTEIN predicted the time dilation of a transported clock in his 1905 paper on relativity[1]. Considering Lorentz transformations between frames in uniform inertial motion with a relative velocity v, he concluded, 'if one of two synchronous clocks at A is moved in a closed curve with constant velocity until it returns to A, the journey lasting t seconds, then by the clock that has remained at rest, the travelled clock on its arrival at A will be $\frac{1}{2}tv^2/c^2$ second slow'. This time dilation effect in special relativity depends only on the relative velocities of the two clocks. The fundamental assertion of special relativity is that there is no local physical means to distinguish an observer in uniform motion from an observer at rest. As Max Planck stated[2], 'the gist of the principle of relativity is the following. It is in no way possible to detect the motion of a body relative to empty space; in fact, there is absolutely no physical sense in speaking about such motion. If, therefore, two observers move with uniform but different velocities, then each of the two with the same right may assert that with respect to empty space he is at rest, and there are no physical methods of measurement enabling us to decide in favour of one or the other'.

This naturally leads to the widely discussed and debated twin paradox[3]. Apparently, it was P. Langevin who presented it as a problem to be resolved in 1911, though it seems to have prior history[4]. In the usual discussions, one of the twins (A) stays stationary on earth, and the other (B) moves out, after initial synchronization of their clocks and perhaps a brief acceleration, in uniform motion (Figure 1). After some duration, the twin B decelerates and stops, turns back, and returns to earth. But for the brief periods of acceleration, this motion is essentially inertial, at uniform relative velocity v. Relative to the stationary twin, the moving twin should age less. But by the assertion of special relativity, formulae for the modification of intervals of space and time involve only relative velocities and the problem can be analysed from either frame, considering a state of rest for that particular frame. So, from the frame of either twin, it is the other one that moves. This generates the paradox, because each clock cannot be behind the other. If time dilation depends only on the kinematics of relative motion with a quadratic dependence on the relative velocity, and if either observer can claim a state of rest and that the clock in the other observer's frame is regis-

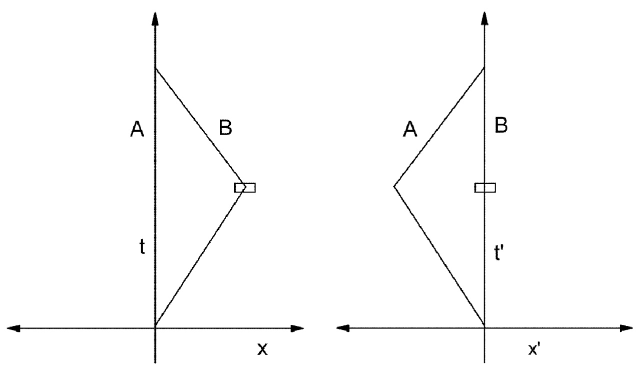

Figure 1. The space-time diagrams for the twin paradox involving clocks A and B. The diagrams are identical when represented from each of the frames, except perhaps at the point of return. Also, the midpoints of the world lines of B contain a short region with acceleration, which is marked with the small rectangle. The proper times are identical if only relative velocities matter and if either observer can claim a state of rest, as they can in the inertial portions of journey.

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tering relatively less time, then the special relativistic time dilation could not be physically real, since permanently imprinted changes in aging should be observer-independent. Which clock aged more is a question that can be settled unambiguously by a direct comparison after a round trip. So, a theory that gives the paradoxical answer that each clock ages less than the other must be a paradoxical and inconsistent theory. This is the statement of the twin paradox.

In the first chapter of his popular account of the theory of relativity, Eddington wrote referring to length contraction and time dilation[5], 'Here is a paradox beyond even the imagination of Dean Swift. Gulliver regarded the Lilliputians as a race of dwarfs; and the Lilliputians regarded Gulliver as a giant. That is natural. If the Lilliputians had appeared dwarfs to Gulliver, and Gulliver had appeared a dwarf to the Lilliputians – but no! That is too absurd for fiction, and is an idea only to be found in the sober pages of science'.

Einstein's resolution

Brief survey of standard resolutions

There are three or four types of resolution in standard textbooks. The least discussed is Einstein's resolution, which is sometimes mentioned without an explicit calculation. The fact that different textbooks resort to different physical reasoning to address the twin paradox is already an indication of confusion and fragility of the standard discussions.

Usually, one insists that the correct calculation is to be done from the inertial frame *A* because *B* accelerated and broke the symmetry of inertial motion. This class of resolution rarely calculates time dilation from the accelerated frame. It draws the space-time diagram with the time in *A* as the vertical axis, and then concludes that the proper time in *A* is more than the proper time in *B* since proper time is a path-dependent quantity. It tacitly assumes that the space-time map in which the path of *B* is shorter (in the Lorentzian way) than that of *A* is the correct one, whereas a space-time diagram drawn with proper time of *B* as the vertical axis will lead to opposite conclusions. The calculation is avoided in the frame *B* because such a calculation is deemed beyond the scope of special relativity, even though such calculations are considered legitimate when one deals with effects like the Thomas precession. Thus the asymmetry in time dilation comes from the asymmetry in accelerations, without any physical reason mentioned, and the correct calculation is the one done relative to the inertial frame. We will come back to this point later.

The second type of resolution invokes a change of line of simultaneity during the turn back of the twin *B*. This became necessary when people posed the twin paradox such that there was no acceleration experienced by the clocks, but merely involved change of inertial frames, by including a third clock *C*. The twin (clock) *B* moves out in inertial motion, and after some time transfers the reading of his clock using a light signal to another clock *C* that is moving inertially in the opposite direction towards *A*. So, *A* and *C* can compare their reading as they pass each other without having to physically turn around the path of one of the clocks. Since no acceleration is involved, some other physical mechanism for the asymmetrical aging needs to be invoked. The space-time diagram in Figure 2 explains the analysis. The diagram is again preferentially drawn from the rest frame of *A*.

All the clocks, *A*, *B* and *C* are in inertial motion. I have indicated the line of simultaneity of *B* and *C* as LS-B and LS-C. These represent a set of inertially moving clocks that are synchronous with the proper time of *B* and *C* respectively. The resolution points out that during the transfer of clock information from the frame of *B* to the frame of *C*, the line of simultaneity has changed, with a discrepancy and advance of time at *A* of Δt. Thus it is suggested that the excess physical time dilation of the *B*–*C* system relative to *A* happens in the short duration of transfer of information from one inertial frame to another.

There is yet another 'resolution' that says that the 'ticks of time' as observed from the frame of the moving twin by receiving light signals sent by the stationary twin are Doppler-shifted differently during the outward and inward journey and there is a small residual second-order difference between these remaining at the end of the journey that correctly accounts for the asymmetrical aging. Thus, in this resolution as well as in the previous one, asymmetrical physical aging has nothing to do with acceleration. We will need to come back to this analysis later,

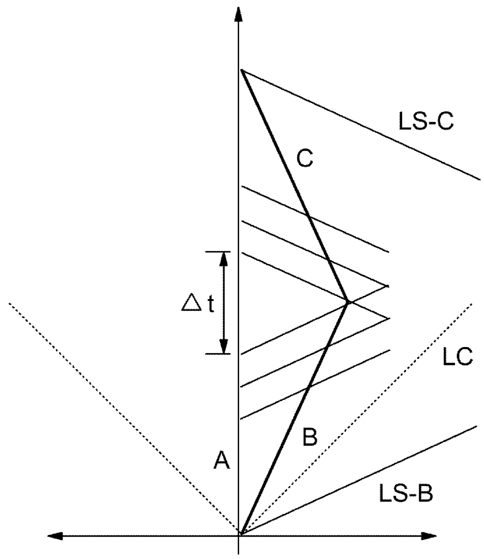

Figure 2. Change of line of simultaneity during reversal of motion of *B*. This amounts to a time advance of Δt on *A*'s world-line. LC indicates the light cone and LS-B and LS-C are the lines of simultaneity associated with worldliness *B* and *C*. The sets of three lines near the point of reversal are parallel to LS-B and LS-C.

after discussing Einstein's resolution, because I think that it is significant that Einstein himself did not invoke any argument of change of simultaneity or Doppler shifts as equivalent to a time lag registered by a clock relative to another. In fact, Eddington, while discussing time dilation in his book[5], had warned against confusing Doppler shift with relativistic time dilation. Next I will discuss Einstein's own resolution that necessarily makes use of the acceleration, and its equivalence to a homogenous gravitational field. Einstein needed the general relativistic physics to resolve the twin paradox in special relativity, and admitted so. This is consistent with his not writing anything that adequately addressed the twin paradox until 1918.

Einstein on the twin paradox

In *The Collected Papers of A. Einstein*[6], there are references to his discussing the twin clock problem in 1911, and to his including its discussion in his lectures in 1914. The problem had been discussed by Einstein around that time, prompted by queries from by E. Gehrcke, later a vehement critic of Einstein's relativity theories[7]. Einstein maintained that accelerations were irrelevant for the amount of time difference between the two clocks, but their presence nevertheless caused the slowing down of clock *B* and not clock *A*. In 1918, almost three years after the completion of the general theory of relativity, Einstein published a paper directly addressing the criticisms of the theory of relativity in the journal *Die Naturwissenschaften*, entitled 'Dialogue about objections to the theory of relativity'[8]. This paper is a dialogue between a critic and an 'adherent' of the physical theory of relativity (relativist), perhaps Einstein himself. None of the textbook discussions, nor any of the papers on the resolution of the twin paradox cite this paper, though it is important because it contained the thoughts of the originator of the special and general theories of relativity on the twin clock problem. Curiously, the discussion starts with a complaint by the critic that none of the relativists had adequately responded to the criticisms of relativity by many in journals. In fact, the critic accuses relativists of 'shirking' the issue. This certainly suggests that Einstein considered that none of the earlier discussions adequately addressed the problem and that it was necessary to respond. Ironically, Einstein's resolution goes against the standard resolutions discussed in textbooks and in most other writings! As the physical cause of the asymmetry he uses the pseudo-gravitational field and the gravitational time dilation of general relativity, after admitting that special relativity is not suitable for resolving the issue due to the fact that one of the twins undergoes accelerations during his trip. However, he does explicitly state that the special relativistic effect can be calculated from the point of view of the intermittently accelerated twin for those portions of the journey that was inertial, and he concludes that during these portions of *B*'s journey, it is the twin *A* who ages less.

Einstein discussed the problem from the point of analysis of each of the frames – the inertial frame *K* in which the twin *A* remains at rest and the twin *B* moves at velocity v_0, and the frame K' in which the twin *B* is at rest, but experiences accelerations. According to him, special relativity alone was insufficient for the entire analysis, since the frames were not equivalent in terms of their inertial properties. From the frame *K*, the analysis is identical to the one usually found in textbooks, leading to the '$tv_0^2/2c^2$' time dilation of *B*, where *t* is the duration of the trip. Then he analyses the situation from the frame K', and agrees that the situation is symmetric except at the point of acceleration. Then he splits the total time dilation into the part that comes from special relativistic kinematics and another part due to the gravitational time dilation that should be present according to the general theory of relativity and the equivalence principle. In the paper he explains to the critic that such effects are expected even in a 'pseudo-gravitational field', and that one need not distinguish between such a field and a true gravitational field in this context. The special relativistic part is indeed a time dilation of $tv_0^2/2c^2$ seconds of the twin *A* relative to *B*. Then Einstein explains that since, 'according to the general theory of relativity, a clock works faster the higher the gravitational potential at the place where it is situated', and since there are homogenous gravitational fields equivalent to the acceleration experienced in the frame K', one should add this contribution in the calculation. He asserts, 'calculation shows that the consequent advancement amounts to exactly twice as much as the retardation during stages of inertial motion. This completely clears up the paradox...'. In passing I note that Einstein admits to the existence of a paradox, if it is accepted that special relativity alone is sufficient to analyse situations where there is no true gravitational field.

Einstein did not include the calculation itself in the paper, presumably because it is a simple calculation that involves only the time dilation formula for two clocks in a uniform gravitational field *g*, separated by a height *h*,

$$\Delta t = tgh/c^2, \tag{1}$$

where *t* is the duration over which the comparison is made. No higher level aspect of general relativity is required. Since the paper itself is not well known and since the calculation is not often discussed in the context of the twin paradox, I reconstruct it here. The derivation here is more detailed than the approximate calculation mentioned in some discussions, and perhaps more detailed than what Einstein had in mind (see below).

If the clock in K' is synchronized with that in *K* during an inertial crossing of the frames, and if the final comparison also is done during an inertial crossing, the only portion of the journey that involves an acceleration is when there is a reversal of motion for the return journey. In Figure 3, this is marked with a set of arrows. In the frame K', it is the twin A that moves and suffers special relativistic time dilation. First, we write the standard time dilation of clock B relative to clock A. (It is clear that when the trajectory is everywhere smooth and symmetric for outward and return journey, then the total time dilation is twice that for half the journey. Only when the reversal of the trajectory is instantaneous this cannot be asserted because one could hypothesize effects associated with the discontinuity in velocity. There is no such effect in special relativity itself though.) If $T/2$ denotes the time it takes for B to reach the point where he starts decelerating, the time dilation of B relative to A for the entire trip is

$$\Delta T_{B-A} = -Tv_0^2/2c^2 - 2\int_0^{t_a/2} \mathrm{d}t\, v^2(t)/2c^2, \tag{2}$$

where the second term accounts for the portion where the velocity is changing due to the deceleration of B. This can be evaluated with $v(t) = v_0 - at$, and $v_0 = at_a/2$, where $t_a/2$ is the time taken to decelerate from v_0 to zero velocity.

$$\Delta T_{B-A} = -Tv_0^2/2c^2 - t_a v_0^2/6c^2. \tag{3}$$

In the usual discussions the second term is small compared to the first term. According to Einstein's analysis, the time dilation of A relative to B is identical if only the kinematical effects are considered. According to Einstein's analysis from B's rest frame, the clock A has indeed a slower rate than the clock B for the portion of the journey that is inertial, and the time dilation of A relative to B is

$$\Delta T_{A-B} \simeq -Tv_0^2/2c^2. \tag{4}$$

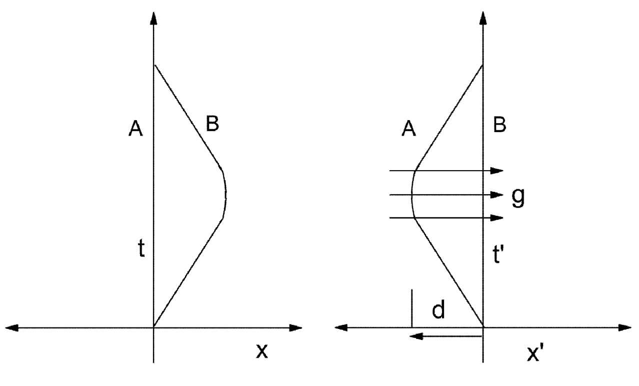

Figure 3. Einstein's analysis of the twin clock problem. (Left) Standard analysis in A's frame and time recorded by the clocks is just the Lorentzian length of the respective world-lines. (Right) Analysis in B's frame, according to Einstein. The special relativistic time is still given by the length of the world-lines and the proper time recorded by A is indeed less than that recorded by B. In addition, there is a gravitational time dilation for a short duration. This is indicated by arrows signifying the presence of pseudo-gravitational field in which B remains at rest and A freely falls.

Thus A ages less relative to B if no gravitational effect is considered. More accurately, considering also the part of the trip where relative velocity changes during deceleration,

$$\Delta T_{A-B} = -Tv_0^2/2c^2 - t_a v_0^2/6c^2. \tag{5}$$

At this stage of the analysis, there is indeed a twin paradox because we get the result that each clock goes slow compared to the other. Einstein proposed to resolve this by adding the time dilation due to a pseudo-gravitational field in B's frame. When B decelerates at rate $-a$, the situation is equivalent to the existence of a global homogeneous gravitational field g as measured in the rest frame of B, in which the twin A falls towards B. At time $T/2$, the separation (equivalent to the 'height' in a gravitational field) between the clocks is $d = v_0T/2$. During the 'free fall', the separation changes as

$$\mathrm{d}(t) = v_0T/2 + v_0t - gt^2/2.$$

Therefore, the gravitational time dilation is

$$\Delta T_g = \int_0^{t_a} \mathrm{d}t\, gd(t)/c^2 = \frac{g}{c^2}(t_a v_0 T/2 + v_0 t_a^2/2 - gt_a^3/6), \tag{6}$$

where $t_a/2$ is the duration of the deceleration that slows down B from a velocity v_0 to zero. Thus the product $gt_a = 2v_0$. We get the gravitational time dilation of A relative to B during the reversal as

$$\Delta T_g = Tv_0^2/c^2 - v_0^2 t_a/3c^2 = -2 \times \Delta T_{A-B}. \tag{7}$$

The clock A, being in a larger (more positive) gravitational potential in the field directed along the line joining A and B, ages faster than B. As Einstein wrote, 'the consequent advancement amounts to exactly twice as much as the retardation during stages of inertial motion'. (It seems from this statement that Einstein had done an approximate calculation, and did not consider the small contribution $-v_0^2 t_a/6c^2$ during the noninertial motion. But there is no escape from calculating the special relativistic time dilation during the noninertial stage as well by assuming that special relativistic formula is applicable to noninertial kinematics. If this term is ignored, the gravitational compensation is imperfect.) The total time dilation of A relative to B is

$$\Delta T = (\Delta T_{A-B} + \Delta T_g) = Tv_0^2/2c^2 + t_a v_0^2/6c^2. \tag{8}$$

Thus A ages more than B by the amount ΔT as analysed from B's frame, which is the same conclusion reached from A's frame as well.

Eddington in his 1920 exposition of relativity[5] suggests a similar cause when he writes that in order for the clocks to be compared again 'the velocity of one of them must be reversed by supernatural means or by an intense gravitational force, so that the conditions are not symmetrical and reciprocity does not apply'. But he did not include any calculation to support this statement. (It is not clear to me what he meant by 'supernatural means' and why a simple reaction thruster and the resulting 'natural' deceleration were not mentioned by Eddington.)

Discussion

The physical flaws in Einstein's resolution

Even before I start discussing some issues of logic and consistency in Einstein's resolution of the twin paradox, the reader would have noticed that Einstein's resolution cannot stand up to the counter examples that do not involve any accelerations! Since Einstein invoked the equivalence principle and a homogenous gravitational field equivalent to the acceleration as the physical cause of asymmetrical time dilation, acceleration is absolutely essential for his analysis to work. But the twin paradox can be stated without acceleration by invoking a third inertial observer. It is surprising that Einstein did not think of this problem and several other logical issues while advancing this resolution of the paradox. First, there is an issue of consistency involved and that too on two counts. It does not seem proper to assert that special relativity was originally consistent with real asymmetric time dilation in the twin clock problem, if a physical effect specific to a different theory that was invented 10 years later was to be used to justify the asymmetric time dilation. Besides, if *B*'s calculation invokes a differential time dilation due to a homogeneous gravitational field, then it is illogical and inconsistent to ignore it in *A*'s calculation. It is true that *A* has no way of determining locally whether there was a homogenous gravitational field present when *B* was reversing in motion, since a homogenous gravitational field cannot be detected while in free fall. *A*'s observation of relative distances is consistent with such a possibility. But, during the comparison of notes, if *A* agrees to *B*'s claim that there was indeed a homogenous gravitational field, then he should also calculate the total time dilation incorporating this fact. This means that *A* would overestimate grossly the time dilation of *B*, and would not find agreement with the observations. It would not be consistent to say that the gravitational time dilation applies only to the calculation done from *B*'s frame since *A* is supposed to use the same physical theory as *B* and since *A* is told that there was indeed a uniform gravitational field for some time, even though he did not detect it due to his free fall. More surprising is the fact that Einstein's resolution does not work in several situations even when there are real accelerations!

Einstein's resolution seems inadequate when multiple clocks are considered, instead of just the two clocks of the twin clock problem, even when one of the clocks undergoes accelerations. Consider two clocks separated by a distance $\vec{d}$ when the acceleration is applied to stop and reverse the moving clock *B*. If the deceleration is $-\vec{a}$, directed along the line joining the clocks in space, there is an equivalent global gravitational field $-\vec{g} = -\vec{a}$. Then it is suggested that the gravitational potential difference between the two clocks in this uniform global field is $\Delta f = gd$ and that the relative time dilation is $\Delta T = Tgd/c^2$, where *T* is the duration for which constant deceleration is applied. Since the decelerated clock is in a deeper potential, the gravitational field being in the direction from clock *A* to clock *B*, clock *B* will age more slowly. Now consider a set of clocks at rest relative to each other and at rest relative to *A*, distributed uniformly in space, and synchronized with *A* (Figure 4). We consider an experiment in which the clock *B* was also synchronized with *A* as it inertially passed *A*. Even though the clock *B* is not simultaneous with all the clocks that are synchronized with A due to their different distances from *B* (since light takes different durations to reach from *B* to each of these clocks), the aging of all these clocks relative to *B* should be identical to the aging of *A* itself. But the gravitational time dilation of the various clocks relative to *B* will be anisotropic, being

$$\Delta T = T - \vec{g} \,.\, \vec{d} \,/c^2. \tag{9}$$

Due to the vectorial dependence on the separation between the clocks under comparison, the gravitational time dilation varies from Tgd/c^2 to $-Tgd/c^2$.

For example, relative to a clock (*D*) whose position vector is orthogonal to the vector $\vec{g}$, the gravitational time dilation relative to *B* will essentially be zero, this clock being in the same gravitational potential as the twin *B* in the global gravitational field. Also, as can be seen directly from the expression for the gravitational time dilation, a pre-synchronized clock that is spatially nearby *B* during deceleration does not have the same time dilation factor as another clock that is far away. Clock *C* in Figure 4 will have approximately half the gravitational time dilation as that of *A*, relative to *B*. Since all the reference clocks *A*, *C* and *D* are at relative rest and synchronized, the physical time dilation of *B* should be identical relative to each of these clocks.

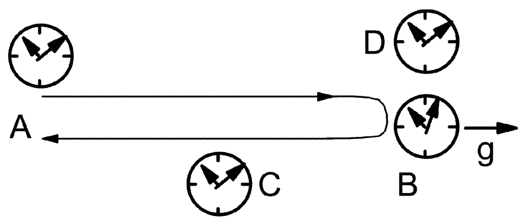

Figure 4. Gravitational time dilation of *B* relative to several synchronized clocks *A*, *C* and *D* that are at rest relative to each other depends on the distance between the clocks as well as on the angle between the direction of gravitational field and position vector of the clock.

A logical fallacy

Einstein's resolution as well as most standard resolutions suffer from a logical fallacy as well. Time is measured by a clock often by marking off the 'phase' of a periodic phenomenon. When two clocks are compared, one is assuming that they are synchronized initially (a constant, known offset in readings), and that their rates of change of phase, or frequency of 'ticks' is the same when they are at rest relative to each other, in the absence of any external physical influence. When they are compared later, the time registered by the clocks can be different only if the rate of the clocks were different during the experiment, before the final comparison. Simply put, a relative difference in the rate of clocks can happen only if the rate of one or both of the clocks is altered due to some reason. It is important to consider the often neglected question – what is the physical cause of the change in the rate of the clocks, and which of the clocks had its rate changed during the experiment? At least the second question needs to be unambiguously answered in the context of the clock comparison experiment.

It is well known from experiments that the rate of a clock, while being affected by motion, does not change due to acceleration. In particular, the rate of a clock in uniform circular motion is the same as the rate of a clock that is in rectilinear motion at the same speed. This means that the rate of the clock *B* does not change in a manner different from what is expected from the usual Lorentz factor modification while decelerating from velocity v to zero. This is the second term in eq. (2), and it is much smaller than the modification of clock rates during the inertial journey. Now, the only possibility to get *A* to age more than *B*, as analysed from *B*'s frame, is to have a change of the rate of clock *A* during the noninertial reversal of *B*. But modification of the rate of *A* that is far away from *B* by altering motion of clock *B* would be violating the restrictions of absence of 'spooky' instantaneous action at a distance in classical relativity!

In summary, we have seen that from a logical point of view, the time registered by two identical clocks that are synchronized initially can be different only if the rate of the clocks changes differently during motion, and one does not see any logical possibility of the required modification of the rate of either clock in any of the standard resolutions, including Einstein's resolution of the twin paradox. I may also note here that a logically consistent possibility is to acknowledge that the rate of a clock is modified according to the standard Lorentz factor with the velocity always relative to the average rest frame of the universe or the frame in which the cosmic microwave background radiation (CMBR) is isotropic, and then there is never a paradox of the clocks. Indeed, the entire voluminous and elaborate writings on the twin clock problem can all be replaced by the single-sentence resolution that the clocks age with Lorentz factors corresponding to their velocity relative to the preferred frame of the matter-filled universe. The answer is always unique, unambiguous and it matches with all known experimental results. Further, it does not discriminate between inertial and noninertial motion and this simplifies and unifies all calculations on clock comparisons, including those required in sophisticated GPS timing. The universe as a preferred frame provides the unambiguous solution to the twin clock problem, and this point is discussed in detail elsewhere[9].

Counter-examples

To clarify the problem with Einstein's resolution, I discuss further variations of the clock comparison experiment that are simpler and more effective. Particularly devastating is a class of thought experiments that involves inanimate and realistic clocks rather than live twins.

Consider an atomic clock *B* that is synchronized with another clock *A* during an inertial encounter. Since the relative velocity between the clocks is known and since the velocity of light is a constant, the time required for a light signal to reach from one clock to another is known to observers in each frame. The interesting and crucial difference between live twins and these electronically controlled atomic clocks relevant for our discussion is the possibility that the atomic clocks can be frozen in their reading at any time, or that they can be switched off and on. In the frozen state, the clock is stopped and no external influence changes its rate or reading. Now consider the situation when *A* sends a signal at his time t_A to *B* to freeze the clock and then decelerate, on receiving the signal. Since *A* can estimate the time at which this signal will reach *B*, he can also freeze his atomic clock at that time (this needs to be known only approximately if the duration of the whole trip is long enough). The readings can be compared after *B* comes to rest relative to *A*, or after a round trip, since the round trip time delays are just twice that of the half-trip when accelerations are smooth and symmetrical. In this experiment, gravitational time dilation has no role to play since the clocks are not running during stages of the journey when there are accelerations! Yet, there has to be asymmetrical time dilation, since it is impossible that each clock is slower than the other (unless one concludes that there is no time dilation at all and therefore the special relativistic time dilation is not real).

This thought experiment shows that all standard resolutions of the twin paradox invoking acceleration or an equivalent pseudo-gravitational field as a physical effect responsible for asymmetric time dilation are flawed, and Einstein's resolution is no exception. Thus Einstein's resolution invoking general relativity, equivalence principle and gravitational time dilation simply does not work, except in the specific case of just two clocks in a very special sequence of motion. Therefore, this resolution of the clock paradox as well as any that rely on some physical effect related to acceleration have to be rejected.

In fact, the possibility of switching off or freezing the clocks makes all the standard resolutions inadequate. Since the experiment can be stopped just before any change of motion for the return trip, and still a direct comparison of the frozen readings can be done, the resolution invoking asymmetry of Doppler shifts between the outward and inward parts of the journey as well as the resolution invoking change of line of simultaneity are untenable.

It is perhaps appropriate to mention briefly another situation where the standard resolutions, particularly Einstein's resolution, give incorrect answers to the clock comparison problem. Consider two clocks in a laboratory that are moving very fast inertially. Then one of the clocks, *B*, is transferred to another laboratory that decelerates and stops, say relative to the reference markers provided by the distant stars or the CMBR. *A* proceeds along its inertial motion. Finally *B* accelerates again such that it comes to rest relative to *A*. In the special relativistic analysis of this problem, it is the accelerated clock *B* that ages less, since the physical situation in a space-time diagram from *A*'s frame is identical to that in the original twin paradox. As Planck mentioned in his lectures, special relativity considers *A* as uniquely at rest. From *A*'s point of view, *B* accelerates away and then proceeds on a journey. Finally *B* decelerates and comes to rest relative to *A*. Applying any of the standard resolutions, including Einstein's gravitational time dilation gives the result that *B* ages less relative to *A*, whereas in this particular problem it is *A* who ages less! This can be verified by analysing the problem from the frame that is at rest relative to the distant stars or the CMBR. We note that the all standard discussions of the twin paradox inadvertently started from both twins being at 'rest' and got answers compatible with experimentally obtained results, whereas if both twins start from high speed inertial motion, the answers are incorrect in the standard analysis. This will be discussed in detail in another paper[10].

Concluding note

Einstein's paper in 1918 was apparently written on advise from friends to respond to the critics of special relativity, in manner that was appropriate for a scientist defending his theory[6]. The very fact that Einstein used the gravitational time dilation predicted by the theory of general relativity, invented ten years after the formulation of special relativity, to justify the asymmetrical time dilation of transported clocks provides some support to the genuineness of the alleged reality of the paradox within special relativity. I have described Einstein's analysis in detail and have shown that there are logical and physical flaws in using the gravitational time dilation to resolve the twin clock paradox. The counter examples discussed here necessitate a reconsideration of the standard analysis of the clock comparison experiments and of the different 'resolutions' of the twin clock paradox. The failure of the accepted views and resolutions can be traced to the fact that the special relativity principle formulated originally for physics in empty space is not valid in the matter-filled universe. Planck's assertion[2] that there is no physical method of measurement of the velocity of motion through space is made void by the various markers available in cosmology, especially the dipole anisotropy of the CMBR. A detailed discussion of this finding along with a consistent formulation of the description of relativity based on the gravity of the matter-filled universe (cosmic relativity) that is fully supported experimentally can be found in Unnikrishnan[9].

This paper is one of a set dealing with the historical and conceptual issues related to the theories of relativity. The motivation is to remove logically inconsistent and inadequate arguments and calculations associated with the discussions of the theory to allow a centenary assessment with clarity.

1. Einstein, A., *Ann. Phys.*, 1905, **17**, 891–921.
2. Planck, M., In *Eight Lectures on Theoretical Physics*, Dover Publications, 1998, p. 112.
3. See any elementary textbook on special relativity for the presentation of the twin paradox.
4. Readhead, M., *Am. J. Phys.*, 1996, **64**, 384–392.
5. Eddington, A. S., *Space, Time and Gravitation*, Cambridge University Press, 1920.
6. *The Collected Papers of Albert Einstein, Volume 7, The Berlin Years: Writings, 1918–1921* (ed. Janssen, M.) (This annotated edition contains extensive commentary and background material in English on the papers, but the papers are not translated. A companion volume of selected translated text is also available), Princeton University Press, Princeton, New Jersey, 2002.
7. I do not address in this article any of the anti-relativity criticisms in the decade of the general theory of relativity. Some of these are historically important and sometimes there are scientifically valid arguments, but they are, by and large, incomplete in logic and in actual calculations. Many of these are invalid criticisms from the point of view of the physical theory. A description of the background for some criticisms may be found in the commentary to reference 6. Later critique by H. Dingle specifically on the twin paradox in the 1960s (detailed in his book *Science at the Crossroads*), is relevant in parts, but misses completely the point that there is indeed an asymmetrical time dilation seen in experiments and that it has to do with motion. The criticism by L. Essen more recently (*The Special Theory of Relativity: A Critical Analysis*, Oxford University Press, 1971) is more accurate, having pioneered the cesium atomic clocks and done clock comparison experiments to test theories, but the suggested ways out are inadequate and fail to address the real physical problem.
8. Einstein, A., *Die Naturwissenschaften*, 1918, **6**, 697. The English translation of the original German version is available in the translated companion volume of *The Collected Papers of Albert Einstein, Volume 7, The Berlin Years: Writings, 1918–1921*, Princeton University Press, Princeton, NJ, 2002, pp. 66–75.
9. Unnikrishnan, C. S., *Cosmic Relativity: The Fundamental Theory of Relativity, its Implications, and Experimental Tests*, gr-qc/0406023.
10. Unnikrishnan, C. S., *Reincarnation of the Paradoxical Relativistic Twins: The Last Episode*, TIFR preprint, to be published.

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