Mind and matter: The question of primacy*

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Two questions

In his book *Ludwig Feuerbach and the Outcome of Classical German Philosophy*, F. Engels asks the question: ‘Which is primary, spirit or Nature?’.

In his book *Mind From Matter*, published posthumously in 1986, Max Delbrück, the noted molecular biologist and geneticist, and former physicist, asks: ‘How can we construct a theory of a universe without life, and therefore without mind, and then expect life and mind to evolve, somehow, from this lifeless and mindless beginning?’

He further asks how such an evolution can lead to a mind ‘capable of elaborating the most profound insights into mathematics, cosmology, matter and the general organization of life and mind itself’.

Neither of these questions makes sense until the terms ‘mind’ and ‘spirit’ have been defined, which neither author has done, and until the word ‘primacy’ is allotted a clear meaning (see note 1). Once, these terms have been defined, the questions become meaningful, and of the two, Engels’ question with ‘primacy’ assigned the meaning ‘temporally prior’, is the more fundamental. For only after Engels’ question, so construed, is given the answer that mindless matter has existed before mindful (or spirited) matter, can Delbrück’s question come up.

It is important to view these questions historically. As Engels points out: ‘... the question of the relation of thinking to being, the relation of spirit to nature – the paramount question of the whole of philosophy – has, no less than all religion, its roots in the narrow-minded and ignorant notions of savagery’.

Anthropologists tell us that in this early phase of human history, apparitions were believed to be the ghosts of dead people, and gods to be disembodied spirits who personified human emotions, and who had to be propitiated by animal and human sacrifice. The spiritual population of the world was made up of much dreaded ghosts and gods.

Unfortunately, Engels does not pursue this early history. Savagery eventually disappeared and during the 7th and 6th centuries BC a great awakening occurred over the Euro-Asiatic landmass. By about that time, the practice of agriculture and the crafts was well established. It is commonplace to us that such practice is successful only because Nature is orderly. But to the ancients the concept of order in Nature was far from obvious and was only slowly assimilated. Sometime in the 6th century BC it dawned on man that the world is a cosmos or ordered whole, that phenomena are subject to general laws, and that these laws are accessible to man and useful in craftsmanship. As Benjamin Farrington, the historian, has written:

For the early Ionians there was no essential difference between natural and technical processes. The claim of the early Ionians that nature was intelligible was based on their view that the practical arts were intelligent efforts of men to cooperate with nature for their own good. The Pythagoreans, the prime movers in the next great philosophical movement, still have the same outlook. Number, for them, is not only the first principle of the heavens, but exhibits its power also in all the handicrafts. (emphasis added).

Thus in the good old days of Pythagoras arose a conception of the world in which human craftsmanship is regarded as dependent on the orderliness of Nature, and the role of mathematics is understood. Heraclitus of Ephesus (500 BC), the great aphoristic philosopher, attributed this orderliness to an immanent rationality that governs the world, which he termed the *Logos* (or *Word*). For Heraclitus, flux and change are paramount features of the world, but they are under the supreme control of the Logos: ‘All things come to pass in accordance with the Logos’.

And this allows the comprehenders of the Logos to utilize the flux for their own good: ‘Wisdom is one – to know the intelligence by which all things are steered through all things’.

Defining the term *mind* as the seat of rationality and orderliness rather than as the seat of self-consciousness, we can say that the Logos doctrine gives the mind (Engels’ ‘spirit’) a regulative or operational (rather than temporal) primacy over the rest of Nature (see note 2). This was Heraclitus’ answer to Engels’ somewhat ambiguously-posed question.

From the Logos doctrine to the scientific methodology

The Logos doctrine was further developed by the Greek and medieval philosophers. It was realized that it is through the agency of the *Laws of Nature*, that the flux in

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*Paper read at a Round Table at the Fifth International Conference on Parallel Computing Technologies, 6–10 September 1999, St. Petersburg, Russia.
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Nature is regulated. Man, to be able to use Nature for his own good, has to discover these laws by the exercise of his brain and hands, and so cooperate with Nature. As Francis Bacon was to say: ‘Nature to be commanded must be obeyed’. From this more developed Ionian perspective, craftsmanship and science are seen as practical–critical activities, involving both hand and brain, and dependent on the extraction of the rationality in the cosmos by the discovery and use of the Laws of Nature.

This view met opposition during the Middle Ages from over-zealous Aristotelian scholastics, who wanted to deduce scientific truth from ‘self-evident’ principles, by brain alone. But the view that thinking as well as making constitute human intellectuality was kept alive by the Franciscans, and it was reinforced during the 16th and later centuries by the development of the scientific methodology at the hands of Galileo, Newton, Francis Bacon, Father Merin Mersenne, Giambattista Vico and Charles Sanders Peirce. It was increasingly recognized that the criterion of scientific truthfulness of a theory is its experimental and observational confirmation, but more fundamentally its efficacy in discovery of the unknown and in the production of better instruments and apparatus, and not in its deducibility from supposedly ‘self-evident’ principles, as the decadent scholastic contemporaries of Galileo had imagined.

The practical side was especially stressed by Francis Bacon, who wrote: ‘That which is most useful in practice might conceivably result by necessity from the truth of that conception; and the sum of these consequences will constitute the entire meaning of the conception. (emphasis added).

What sort of intellectuality is meant in the qualification ‘intellectual’ appearing in Peirce’s statement? This question had been answered in large measure again by the Pythagoreans long ago: fundamentally, the intellectuality has to come from pure mathematics. This was demonstrated in the mathematical demarcation of the notion of ‘pure tone’ in acoustics, c. 500 BC, and of the concepts ‘momentum’, ‘kinetic energy’ and ‘pressure’ in the 17th century, and has been demonstrated again and again thereafter. The great discoveries in electromagnetism and wave mechanics, associated with the names of Maxwell and de Broglie, rested on the pursuit of deep mathematical analogies. It will suffice in this regard to merely quote the words of the great physicist P. A. M. Dirac:

I learnt to distrust all physical concepts as the basis for a theory. Instead one should put one’s trust in a mathematical scheme, even if the scheme does not appear at first sight to be connected with physics. One should concentrate on getting an interesting mathematics.

How it may be asked can the ideal concepts of pure mathematics, sub specia aeternitatis, have practical efficacy? Is there not a conflict between the assertions of Peirce and Dirac? Absolutely not. In fact, the pragmatist and mathematicist or Logosist positions are, as Plato understood, cf. Plato, fully concordant, for as Peirce trenchantly maintains:

...if pragmatism is the doctrine that every conception is a conception of conceivable practical effects, it makes conception reach far beyond the practical. It allows any flight of imagination, provided this imagination ultimately alights upon a possible practical effect; and thus many hypotheses may seem at first glance to be excluded by the pragmatical maxim that are not really so excluded. (emphasis added).

An early illustration of the practical role of the mathematical imagination is provided by the resolution of the impasse in post-Copernican astronomy, that came from the insistence that planetary motion be circular with constant speed. The astronomers asked: ‘The circle being the most perfect of curves, how can the heavens (which “declare the glory of God”) exhibit non-circular, accelerating motions? But the kinematically oriented mind of Johannes Kepler saw it differently. It is the “music of the spheres” that the Pythagoreans had venerated, and the circle, conceived kinematically, yields a dull extended pure note, far from perfection from a musical standpoint.’ Here are Kepler’s own words:

He who speaks of music, has in mind variety, not monotony. Therefore, ought not a God, who is a musician, to assign to each of the planets its own proper musical phrase instead of one single ‘note’? and from these individual phrases to develop in the course of time a polyphonic and contrapuntal harmony? To achieve this, what matter if the strict bounds of geometrical ratios be transgressed? what matter if the results of natural necessities be subjected to the physical mechanism of eccentric motions?

By this imaginative flight, Kepler was soon led to the discovery of elliptic orbits, and to the completion of his other laws of planetary motion. From this work of Kepler science got its greatest boost since the days of Plato, and the way was paved for Newton and his successors.

Another illustration, more germane to this paper, is offered by de Broglie’s bold idea of associating with each material particle a wave, derived solely from his musings on William Rowan Hamilton’s unification of the Maupertius principle of least action in mechanics with Fermat’s principle of least time in optics, and Einstein’s concept of light quanta. This idea has proved to be overwhelmingly
fruitful. It underlies the construction of the electron microscope, one of the finest instruments produced in this century, quite apart from its providing the very foundation of the wave mechanics that govern phenomena at the atomic level.

To sum up, the important pragmaticist principle of the scientific methodology is reinforced by the mathematical ideality of empirically inspired conceptualization.

Can mindless objects obey mathematical laws?

The mathematical ideality of empirical scientific conceptualization has interesting implications. Consider the moon’s gravity. A marble released near the moon’s surface will receive an acceleration $GM/r^2$ towards the moon’s centre, according to Newton’s law of gravitation, $M$ being the moon’s mass, $r$ the marble’s distance from the moon’s centre, and $G$ the gravitational constant. Does not the impartation of this exact acceleration suggest that the moon takes some accounting of what $G$, $M$ and $r$ are, of their values, and of how they are combined together, and so has a mental capacity? We are using the word ‘mind’ from which ‘mental’ is derived, to mean the source or seat of intelligent reckoning. But Newton’s law is an approximation to Einstein’s more accurate general law of gravitation, involving much harder mathematics, and this in turn, is only an approximation to the unknown true law of gravitation. This true law will, if anything, be even more mathematical, for again to quote Dirac:

It seems to be one of the fundamental features of nature that fundamental physical laws are described in terms of a mathematical theory of great beauty and power, needing quite a high standard of mathematics for one to understand it. You may wonder: why is nature constructed along these lines? One can only answer that our present knowledge seems to show that nature is so constructed. We simply have to accept it. . . . Our feeble attempts at mathematics enable us to understand a bit of the universe, and as we proceed to develop higher and higher mathematics we can hope to understand the universe better.

It is therefore fair to suppose that the true law of gravitation, which the moon obeys, is exceedingly mathematical. The question this raises (of which neither Engels nor Delbrück seem to be aware) is the following: ‘Can a completely mindless object obey highly mathematical laws?’

If this question is answered negatively, then the moon has at least some semblance of mentality. In this case, Delbrück’s premise that all matter before the advent of life is mindless is atrophied? If on the other hand, the question is answered affirmatively, then there are mindless objects such as the moon, which obey highly mathematical laws. Presumably all non-biological objects fall in this mindless category, among them those that perform intelligent tasks, such as pocket calculators, electronic computers and automaton like the ‘Deep Blue’, which beat Kasparov at chess. But then where does the ‘mindful’ category begin? If the (non-biological) automaton ‘Deep Blue’, obedient to mathematical laws, is mindless, then on what grounds can Kasparov, whom it defeated, be deemed mindful? It would seem that the behaviourists are correct, and Delbrück’s belief that man, who evolved from mindless matter is mindful, is unfounded; for man’s intellectual abilities may not necessitate his having a mind, any more than the automaton Deep Blue’s intellectual abilities.

In short, our choice can only be between two alternatives – (1) The immanent rationality in the cosmos, which manifests itself in the Laws of Nature, infects all objects with a mentality, however limited (extreme Logosist position), or (2) No object has a mind, i.e. mind is a pseudo-concept (Extreme behaviourism).

The non-behaviourist has to accept the first alternative. If you believe that Kasparov has a mind, then you must believe the same for the automaton ‘Deep Blue’; if this automaton is mindless, then so is Kasparov. In Delbrück’s book there is no mention whatever of the position of the Behaviourist School, which denies the authenticity of the concept of mind, nor of the contrary position of Charles Sanders Peirce, according to whom there is a continuum of mind, and ‘matter is just mind hidebound in habit’.5

It should be noticed that the question of evolution is still valid, for even if all things have a mind, the vast difference between the mind of the moon and the conscious, discerning and creative mind of an Einstein or a Mozart needs accounting. This developmental problem also remains for the behaviourist. Thus Delbrück’s question with the more careful formulation: ‘How do terrestrial life and highly developed minds evolve from the bare (and lifeless) mentality that primordially pervades everything in the universe?’ remains a very important question, that demands an answer from the Logosist. A corresponding evolutionary question for the behaviourist also demands an answer.

The limitation to the two alternatives mentioned earlier places Engels and all so-called dialectical materialists in a quandary. For they condemn the denial of all mind as ‘vulgar materialism’ (Mind exists: it is ‘matter that thinks’); but they also condemn the Logosist position as ‘idealistic’ and therefore wrong! St. Petersburg is the birthplace of behaviourism. As Wiener has emphasized10, the conditioned reflex marked the beginning of scientific psychodynamics (see note 3). The reflex made it clear how a good deal of purposeful activity occurs without the exercise of intelligence. However, this is a far cry from total behaviourism. If Academician Ivan P. Pavlov could be resurrected, we could ask him point blank: ‘Do you accept the alternative 1 or the alternative 2, or do you believe that there is a third alternative?’
Leibniz, the patron-saint of Cybernetics

In the 17th century, Gottfried Wilhelm Leibniz, the great genius, gave the Logosist position a tremendous boost by affirming that the fundamental units of which the world is made are pointes endowed with mentality, which he called monads.

Thus according to monadology, all things are infected with mentality, and so no object in the world can be totally mindless. Thus from the standpoint of monadology, the third question is vacuous. As for behaviourism, monadology denies its meaningfulness. Behaviourism is often described as a theory in which man is deemed to be a machine. The behaviourists never define exactly what a ‘machine’ is. Whatever it is, monadology teaches that it cannot be mindless. Thus if man is a machine, then man is mindful. We see that without careful redefinition, behaviourism collapses.

Clear definitions of machine were given only in the post-World War II years by Norbert Wiener and W. Ross Ashby, among others. As Wiener has written:

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In his paper Ashby defines a machine, more narrowly, as a discrete time-transducer with internal states, such that sn and en are the internal state and external conditions at instant n, respectively, then sn+1 is computable from sn and en.

Leibniz, as is well known, was also a pioneer in the field of computing. In 1671 he built a computer that superseded the ancient abacus as well as the 1642 adding machine of Pascal. But his vision went far beyond. He envisioned for the future the possibility of a symbolic logic (the calculus ratiocinator) operable by machine (the machina ratiocinatrix), which would handle mankind’s logical problems. These hopes of his have been fulfilled today in large measure. Leibniz furthermore envisioned a more inclusive ars characteristic reformulated into a computer which would be able to handle mankind’s problems. These hopes of his have been fulfilled today in large measure. Leibniz furthermore envisioned a more inclusive ars characteristic, i.e. a general theory of signs (semiotic, in modern terminology) which would subsume the above. Here too his hopes are materializing, thanks to the pioneering work of C. S. Peirce, R. Carnap, N. Chomsky and others. Not unjustly did Wiener call Leibniz, ‘the patron-saint of Cybernetics’.

Limitations of Leibniz as a scientist

In his understanding of science, Leibniz, his genius not withstanding, fell short of his great predecessors and contemporaries such as Galileo, Pascal, Huygens, Newton, Boyle and Vico. Although he built a computer and took great interest in the new field of microscopy, he failed to grasp the epistemological significance of practice, and of a truth-criterion based on practice. Instead he held to the Aristotelian tenet that validity in science rests on deductibility (see note 4) from self-evident principles – a belief based on the misconception that Euclidean geometry is paramount.

Furthermore Leibniz could not shake off the Cartesian spell of his day. Fundamentally, his doctrine of pre-established harmony merely condensed in time the continual Divine intervention (doctrine of occasionalism) that Descartes invoked to maintain the parallelism between mind and body. Leibniz’s system thus rested on an intrinsically unscientific premise, transgressing the ambit of natural law.

As Bertrand Russell has pointed out, Leibniz’s comprehension of science was impaired by his inability to go beyond the subject–predicate logic of Aristotle, and to reach a logic of relations. Sensory prosthesis, e.g. of deafness, rests on the analysis of auditory relation-structure, and not on a metaphysical study of a being called Deafness. Likewise, the planet Neptune was discovered in 1846 by pursuing the mathematics of gravitation, and not by delving into the mystery of action-at-a-distance. Science advances in this manner, by leaning on experimentation and mathematical analysis, and by eschewing ontological and metaphysical questions. Leibniz, who was more of a rationalist philosopher than a scientist, still lived in a world in which ‘the noun was hypostasized and the verb carried little or no weight’, to use Wiener’s words, and brought metaphysics into his discourse. He wrestled with the relationship between the whole and its parts unsuccessfully, since his understanding of the infinite, in particular of a continuum, was flawed.

We must remember that Leibniz, a founder of the infinitesimal calculus, lived half a century before Lobatchevsky, and a century before Dedeking, Cantor and Peirce, all of whom stood on his shoulders. Understandably, Leibniz stumbled on certain matters, geometric, transfinite and relational. But his far-reaching mind enabled him to compensate for this time-lag, and say some very valuable things. Thus Leibniz’s Monadology, its metaphysical foliage not withstanding, is still of scientific interest. Speaking of his book The Human Use of Human Beings, Wiener was to write: ‘Leibniz dominated ideas of communication, is, in more than one way, the intellectual ancestor of the ideas of this book, for he was also interested in machine computation and in automata’.

The Monadology, apart from its firm disposal of the question of primacy, lays out a conceptual structure that
has a certain affinity to that of quantum mechanics. It also bears on the question of intelligent automata.

The Monadology of Leibniz

Leibniz’s Monadology contains 90 numbered items, these ranging in size from a sentence or two, to a paragraph or two. It is comprehensive, and covers the nature and classification of the monads, their relation to the uncreated monad, God, the nature of truth, the laws of contradiction and sufficient reason, the doctrine of pre-established harmony, birth and death, the relation of body and soul, and the reconciliation of final and efficient causes. What follows is a very brief summary of the portions (with numbers cited) relevant to this paper.

The simplest units from which the world is made are points endowed with mentality called monads (no. 1). They are all created by God, and are immutable. Being a dimensionless point, a monad has no ‘windows’, i.e. inlets and outlets through which it can communicate with the rest of the world (no. 7). But each monad has its individual qualities, and so no two monads are equal (no. 8). Moreover, the quality of each monad changes by an internal principle (no. 11). The monads have a self-sufficiency that makes them the only source of their internal activity. Leibniz speaks of them as incorporeal automata (no. 18) (see note 5). The Monadic activity that brings about change is called appetition.

Though the monad has no parts, it has multiplicity (no. 13). Since during a change some qualities are altered and others are not, there is a multiplicity of qualities. The changing condition of the monad is called its perception of the world—a blurred awareness of it, to be distinguished from full consciousness, called apperception (no. 14). (In A. N. Whitehead’s terminology, we may say that the bare monad ‘prehends’ but does not ‘apprehend’.) Each monad, by its perception, represents the world around it from its own standpoint, which may be blurred and limited, or clear and long-range (nos 56 and 57). The complete description of a monad consists of a full record of the sequence of its perceptions.

In the monadic terminology, the predicates blurred, ‘material’, ‘passive’ go together, as do the predicates ‘clear’, ‘spiritual’, ‘active’ (nos 49–52). In his paper read at the Norbert Wiener Centenary Congress in 1994, Gale gave a diagram to illustrate this (Figure 1). The first set of predicates, with ‘material’ singled out, is represented on the y-axis. The second set, with ‘spiritual’ singled out, is represented on the x-axis. The typical monad may be characterized by its coordinates (x, y), where x + y = 1 and 0 ≤ x, y ≤ 1; x, y are the degrees of spirituality and materiality. The fully spiritual monad, viz. (1, 0), would be God.

Monads with clear perception and possessing memory are called souls (no. 19). Memory provides the soul with a consecutiveness (no. 25). A bare monad, like a soul in a dreamless sleep, is in a state of stupor. In the animals perpetual stupor is prevented by the bodily sense organs, which collect numerous stimuli from outside and keep the animal alert (nos 24 and 25). Man differs from the animal in that the concatenation of his perceptions is governed not just by memory, but by a rational soul, which gives him knowledge of necessary and eternal truths (no. 29).

Among the monads that constitute a material body, the one with the clearest perception, determines the character or type of the body, and is called the dominant monad (no. 70). The others are called subordinate monads. If the dominant monad in the body is a bare monad with unconscious perceptions, then the body is inorganic. If the dominant monad in the body is a little clearer, then the body is a plant. Bodies in which the dominant monad has consciousness and memory, are the animals (no. 70). In man the dominant monad is more than this: it is the rational soul.

A material body without a semblance of mentality is impossible, since such a body would be devoid of monads. Among the bodies there is no sharp dividing line between the inorganic and the living. Each body has a certain degree of ‘organic unity’.

In virtue of their appetition all bodies are in constant change, either unfolding, i.e. passing from confused to more distinct perceptions, or infolding, i.e. passing from distinct to more confused perception. Thus the body of a monad is in perpetual flux ‘like a river’: as time passes its dominant and subordinate monads change (no. 71).

There is no absolute birth, i.e. direct implanting of a soul into a body, and there is no absolute death, i.e. complete severance of soul from body, except in so far as it comes from direct divine intervention (no. 73). The only soul without a body is God (no. 72).

Leibniz, the Cartesian, was a determinist. On the hierarchical causal aspect of monadology, the following words of Wiener are better suited to this audience than those from the Monadology:

Each of them [monads] lives in its own closed universe, with a perfect causal chain from the creation or from minus infinity in time to the indefinitely remote future; but closed though they are, they correspond one to the other through the pre-established harmony by God.
Leibniz compares them to clocks, which have so been wound up as to keep time together from the creation for all eternity. Unlike humanly made clocks, they do not drift into asynchronism; but this is due to the miraculously perfect workmanship of the Creator.

Thus Leibniz considers a world of automata which, as is natural in a disciple of Huyghens, he constructs after the model of clockwork. Though the monads reflect one another, this reflection does not consist in a transfer of the causal chain from one to another. They are actually as self-contained as, or rather more self-contained than, the passively dancing figures on top of a music-box. They have no real influence on the outside world, nor are they effectively influenced by it. As he says, they have no windows. The apparent organization of the world we see is something between a figment and a miracle. The monad is a Newtonian solar system writ small.

The import of the Monadology for contemporary science

There are inconsistencies in Leibniz’s monadology, to which several commentators have drawn attention. To cite a couple, different perspectives on the same reality, such as are discussed in Bishop Berkeley’s writings, come from differences in the nature, location and environment of receptor organs. How can an insulated point-soul, devoid of such organs, have its own perspective on the world, i.e. how can it ‘mirror the universe’? The answer that the mirroring comes from a pre-established harmony is specious, scientifically speaking. Secondly, Leibniz denied without justification a conclusion latent in his system, to wit: the universe itself is a body with its own dominant monad.

Nevertheless, Wiener saw in Leibniz’s monadology many valuable insights for modern science. He accordingly undertook to remove the monad from its metaphysical abode, and then examine how it tallied with the elementary particles of modern physics.

There are fundamental divergencies between Leibniz’s monadic ideas and those of modern physics. According to Leibniz, the world is a plenum and matter is not atomic, which is in accord with relativity theory, but in sharp contrast to what quantum physics teaches. Secondly, the Leibnizian cosmos is strictly deterministic, whereas the cosmos of modern physics is stochastic. Thirdly, whereas any two monads are unequal, two electrons (or other particles) with the same spin, are indistinguishable, and therefore equal, apart from location.

Nevertheless, in his 1932 paper, Wiener suggested how the blurring of the perception of Leibnizian monads may be squared with the breakdown of determinism in quantum mechanics. Referring to the adaptations of Kaluza’s five-dimensional theories to wave mechanics, due to D. Klein and V. Fock in the mid-1920s, Wiener wrote:

Thus, each electron possesses its own world of dimensions, which mirrors the many-dimensional universe of perfect cause and effect in an imperfect, four-dimensional, non-causal image. It is surely not fanciful to see in this a parallel to the Leibnizian monads, which live out their existences in a self-contained existence in pre-established harmony with the other monads, yet mirror the entire universe. (emphasis added).

Dwelling on the Leibnizian proposition that the existing universe is the best among all possible universes, the best universe thus receiving probability 1 and each of the others probability 0, Wiener pointed out how a more equitable probability distribution would suggest to us the statistical mechanics of J. W. Gibbs. Tentatively, Wiener pointed out other such connections between Leibnizian and modern ideas.

In 1934 came a speculative paper by J. B. S. Haldane, the famous geneticist and versatile scholar, entitled ‘Quantum mechanics as a basis for philosophy’. Although Leibniz is never mentioned, this paper had a tonic effect on Wiener’s reflections on Leibniz; see Wiener.

The features of Haldane’s thought that bring it closest to the monadology are his two theses:

(A) From the standpoint of modern science, the world neither comprises matter in the strict Newtonian sense, nor comprises ideas in the strict Platonic sense. Rather, the Newtonian and the Platonic worlds are ideal limits of the existing world, as the mass tends to infinity or zero, respectively.

(B) Quantum mechanics eradicates the sharp separation between the inorganic, the living and the mental.

With regard to (A) we have only to take a hyperbolic-shaped curve, and write ‘Platonic world’ and ‘Newtonian world’ at the termini of the x and y axes, respectively, in Figure 1 in order to convert this monadic diagram into one that illustrates Haldane’s thesis (A) (see Figure 2). As for the thesis (B), it is of course consonant with the thrust of the monadology that each body has a certain degree of organic unity.
Haldane on the mind of a material system

The wave–particle duality of modern physics is central to Haldane’s hypothesis. In a nutshell this hypothesis asserts that the de Broglie wave of an elementary particle serves as its nascent mind, and that both life and developed mind are resonances of the de Broglie wave-system associated with material systems. The Appendix gives requisite ideas of (A) wave–particle duality, (B) resonance and degeneracy, and (C) the transparency of potential barriers, all of which come up repeatedly in Haldane’s argument.

To elaborate on these matters, we can do no better than quote the following summary from Gale’s paper, and then let Haldane speak:

The main thrust of Haldane’s paper concerns the new light that modern physics sheds on both the old issue of vitalism, i.e. whether the laws of physics can explain biological phenomena, and on the mind–brain problem of psychology. It is Haldane’s view that the new physics, by virtue of its recognition of the wave–particle duality of matter and energy, is rich enough to explain the phenomena of life and mind.

The new properties of matter that Haldane finds most relevant are:

(a) The complete identity, apart from space–time location, of all elementary particles of a given type, e.g. of all electrons.

(b) The non-localizability of electrons stemming from its de Broglie wave, reminiscent of the non-localizability in either space or time of mental events such as thoughts, which fade in and out.

(c) The positive probability that the elementary particles will leak through a potential barrier. This allows the possibility of attaining a goal (teleology).

(d) The ability of atoms and molecules of self-repair; an atom which has lost one of its electrons quickly picks up a new one, again reminiscent of biological organisms.

(e) Complimentarity: the failure to simultaneously measure certain quantities, again reminiscent of psychological facts such as the impossibility of simultaneously being angry and making an introspection about it.

On Haldane’s important speculations on the mind, it will suffice to quote some salient aspects of his thought, as presented in his paper.

If mind is to be regarded as expressive of the wholeness of the body, or even of the brain, it should probably be thought of as a resonance phenomenon, in fact part of the wave-like aspect of things. In a degenerate system degrees of freedom are lost because certain periodic systems oscillate together instead of independently. This resonance gives rise to various observable phenomena. It is responsible for certain terms in the energy of a material system. As the resonators are removed from one another, the energy falls off very rapidly. If mind is a resonance phenomenon we do not as yet know what the resonators are. They might conceivably be molecules on the one hand, or more probably whole cells or large parts of them on the other. For of course cells in the nervous system undergo periodic electric disturbances, and Lapicque’s idea of isochronism does not perhaps differ essentially from resonance. In any case the amount of energy concerned in mind must be exceedingly small.

If mind is a resonance phenomenon it is at once clear why it cannot be definitely located, either in space or time, though it is obviously enough connected with definite events in a definite material structure. The smaller the mass (or energy) of a mental event, the greater may we expect its indeterminacy to be. Thus the continuous character of our sensory experience becomes intelligible.

To escape from metal into a gas at a lower potential, an electron must pass through a potential barrier. It does not possess sufficient energy to do so according to the laws of pre-quantum physics. But this does not in fact prevent its escape. From the point of view of wave mechanics we can say that the electron’s wave system leaks through the barrier until the probability that the electron will be found outside it becomes large. This is a reduction to mechanical terms of the electron’s apparently purposive conduct, and enables us to predict this conduct on a statistical scale. In just the same way an alpha particle escapes from the nucleus of a radioactive atom. Its wave system permits it to act with reference to the field of lower potential outside the nucleus, which it will enter after traversing the barrier.

It is suggested that man also has a ‘wave system’ which enables him to act with reference to distant or future events, this system being his mind. Of course no more in man than in the alpha particle does the future actually determine the present. We make our plans for the future on the basis of data now available. But on the basis of these data we deliberately undergo temporary unpleasure or pain for the sake of greater future pleasure, to make a simple instance analogous to the behaviour of the alpha particle. The analogy appears to be close enough to make it unnecessary to postulate the emergence of purpose as a qualitatively new phenomenon associated with the mind. The electron can penetrate its potential barrier because its wave system effectively extends beyond it. The wave systems of the individual water molecules do not extend beyond the cistern to a sufficient extent to allow them to jump out of it on an observable scale. Nor does a volume of liquid as a whole possess the requisite wave system. But a man, or even a mouse, does.
This at once raises the very interesting question how far the mind, regarded as a wave system, could be said actually to extend out into space so as to comprehend distant objects. There is certainly no formal objection to this view, since the de Broglie waves of any particle are supposed to be omnipresent (see note 6). There would seem to be at least a possibility of overcoming the objections to a realistic doctrine of perception which Berkeley and his followers have found in the physiology of the sense organs. In some sense the mind may actually extend out to physical objects. When I perceive a spiral nebula some millions of years in the past there is perhaps a physical sense in which this nebula (separated from my retina by an interval of measure zero in space–time) is actually part of my mind.\(^{18}\)

Was Haldane, for many years a member of the British Communist Party, and sometime editor of the London Daily Worker, a materialist or an idealist? This is a naive question. The expression of his thought is weighted perhaps in favour of matter. But, as he was quick to point out, matter obeys the wave mechanics, and is not devoid of mentality. As to whether his theory is ‘materialist’ or ‘idealist’, Wiener gave a definitive answer when he wrote:

\[
\text{I can see no essential difference between the materialism which includes soul as a complicated type of material particle and spiritualism which includes material particles as a primitive type of soul.}\]

\(^{19}\)

Indeed, one contribution of Haldane’s inquiry has been to show that the old controversy between idealism and materialism, ‘the two great camps’, to which Engels attached such great importance, is almost entirely bogus.

Time does not permit us to go into other facets of Haldane’s thought. We would refer the reader to Haldane’s paper\(^{18}\) and to his 1939 book The Marxist Philosophy and the Sciences\(^{20}\).

Haldane’s ideas have been neglected in Western Europe and in the United States. True, they remain highly speculative. But Haldane was a very great thinker. It would therefore be very good if the younger generation of scientists in Russia could pursue his line of thought, for to me it seems to be on the truthful trail.

**Appendix**

**Requisite wave mechanics**

It is worth considering the curious vocabulary used by the pioneers who founded wave mechanics. For instance, in his fundamental paper of 1926, Max Born, commenting on the widely different interpretations of the wave function proposed by Heisenberg and Jordan on the one hand, and Schrödinger on the other, wrote:

\[
\text{Neither of these two conceptions appears satisfactory to me. I should like to attempt here to give a third interpretation and to test its utility on collision processes. In this attempt, I adhere to an observation of Einstein on the relationship of wave field and light quanta; he said, for example, that the waves are present only to show the corpuscular light quanta the way, and he spoke in this sense of a ‘ghost field’. This determines the probability that a light quantum, the bearer of energy and momentum makes a certain path; however, the field itself has no energy and no momentum.}
\]

\[
\ldots\text{With the complete analogy between a light quantum and an electron, however, we shall consider the formulation of the laws of the motion of electrons in a similar manner. And here it is obvious to regard the de Broglie–Schrödinger waves as the ‘ghost field’ or, better, ‘guiding field’}.\(^{21,22}\). \text{(emphasis added)}
\]

The words ‘ghost field’, ‘guiding field’, and de Broglie’s own term ‘pilot wave’, suggest that these waves play a role in physical processes rather like that of our thoughts in guiding our actions. It was Haldane’s speculation that this vague analogy could be made the basis of a sound theory of associating a mind with every material system that we sketched in Figure 2.

As for the reality of the wave function \(\psi\), the opinion that \(|\psi|^2\) is somehow real but \(\psi\) is a ‘mathematical fiction’, is dismissed by Born, who wrote: ‘I personally like to regard a probability wave, even in \(3N\)-dimensional space, as a real thing, certainly as more than a tool for mathematical calculation.\(^{23}\)

**A. De Broglie waves of a material system:** We must first define the de Broglie wave for the simplest material system, viz. a single free, spinless particle, uninfluenced by any field of force.

De Broglie believed that Einstein’s association of light quanta (particles) with light waves should have its counterpart in a similar association of waves with material particles. He accordingly began with the assumption that every material particle has associated with it a periodic process, and then let relativity theory take over.

Suppose that the periodic process associated with a particle of rest mass \(m_0\) (and therefore rest energy \(E_0 = m_0c^2\)) has the frequency \(v_0\), called the rest frequency of the particle. Special relativity teaches how this periodic process will appear in an inertial frame of reference (FR) \(S\) with respect to which the particle is moving uniformly.

For let the particle move with constant speed \(u\) along the \(x\)-axis of an inertial FR \(S\), and let \(S'\) be a FR with origin fixed to the particle, and having the same \(x\)-axis as \(S\). Let the \(y\)-axes of \(S\) and \(S'\) be parallel. The rest frequency \(v_0\) of the particle gives us a simple harmonic motion (SHM) along the \(y\)-axis of \(S'\), viz.

\[
y(t') = \cos 2\pi v_0 t' = \text{Re } e^{i\pi v_0 t'}.\]
But by special relativity,
\[ t' = \frac{1}{a} \left( t - ux/c^2 \right), \quad \text{where} \quad a = \sqrt{1 - u^2/c^2}. \]

Hence with respect to the FR \( S \), the SHM spreads out as a wave, its equation taking the form
\[ \psi(t,x) = \text{Re} \ e^{2\pi i \nu t/a} \left( t - ux/c^2 \right). \tag{1} \]

Equation (1) shows that the \( \psi(t,x) \) is a plane sinusoidal wave in the \( x \)-direction, and that its frequency \( \nu \) with respect to \( S \) is
\[ \nu = \frac{\nu_0}{a} = \frac{\nu_0}{\sqrt{1 - u^2/c^2}}. \]

But special relativity also teaches that the particle’s energy with respect to \( S \) is
\[ E = mc^2 = \frac{m_0}{\sqrt{1 - u^2/c^2}} c^2 = \frac{E_0}{\sqrt{1 - u^2/c^2}} = \frac{E_0}{a}. \]

We see at once that \( \nu \nu_0 = 1/a = \nu_0/E_0 \), whence
\[ h' := \frac{E}{\nu} = \frac{E_0}{\nu_0} = \frac{m_0 c^2}{\nu_0}. \tag{2} \]

Thus \( h' \) is independent of the speed of the particle relative to \( S \). Moreover, \( h' = ET \), where \( T = 1/\nu \) is the period of oscillation, and therefore \( h' \) has the dimensions of action (= energy \( \times \) time). Now as \( m_0 \to 0 \) and \( u \to c \), our particle turns, so-to-speak, into a photon governed by the Planck–Einstein equation \( E/\nu = h \), where \( h \) is Planck’s elementary quantum of action. These heuristic considerations strongly suggested to de Broglie that \( h' \) is just Planck’s constant \( h \).

de Broglie thus derived from eq. (2) the sharpened hypothesis that with every material particle of rest mass \( m_0 \) is attached a periodic process of rest frequency
\[ \nu_0 = \frac{1}{h} m_0 c^2, \quad h := \text{Planck’s constant}. \]

It readily follows from this that the frequency \( \nu \) and wave number \( k \) of the sinusoidal wave eq. (1), are given by
\[ \nu = \frac{1}{h} E \quad \text{and} \quad k = \frac{1}{h} \frac{p}{\nu}, \]

where \( p \) is the momentum of the particle relative to \( S \). Eq. (1) thus represents a real plane sinusoidal (monochromatic) wave of frequency \( E/h \) and wave number \( p/h \). Its speed is
\[ V = \frac{E}{h} \frac{h}{p} = \frac{mc^2}{mu} = \frac{c^2}{u} > c. \tag{3} \]

It is convenient to omit the prefix ‘Re’ in eq. (1), i.e. to deal with the complex sinusoidal wave. We may sum up as follows:

**Theorem (de Broglie, 1924)** – With every material particle moving on the \( x \)-axis of an inertial FR with constant speed \( u \), is associated a complex sinusoidal plane wave in the \( x \)-direction,
\[ \psi(t,x) = e^{2\pi i (\nu' - kx)}, \]

whose frequency \( \nu \), wave number \( k \) and speed \( V \) are given by
\[ \nu = \frac{1}{h} mc^2 = \frac{E}{h}, \quad k = \frac{1}{h} mu = \frac{1}{h} p, \quad V = c^2 lu, \]

where \( m, E, p \) are the mass, energy and momentum of the particle relative to \( S \). This wave is called the de Broglie wave of the particle. (It travels faster than light.)

It is easily seen that the speed of the de Broglie wave depends on its frequency, i.e. that the propagation is in a dispersive medium. There is a dispersion function \( g : \nu = g(k) \) connecting the frequency \( \nu \) and wave number \( k \). It has been known since the days of Lord Rayleigh (1850) that in dispersive propagation the bulk of the energy in the waves is concentrated in a ‘wave packet’, and that the speed of the packet is \( g'(\alpha) \alpha \) being the central wave number in the packet. A simple calculation shows that for de Broglie waves, \( g'(\alpha) = \alpha \). Thus, the speed of the de Broglie wave packet is equal to the speed of the particle.

Since the wave packet generally spreads out with the passage of time, it follows that in wave mechanics the particle is not precisely locatable.

de Broglie next showed that for a particle of rest mass \( m_0 \), moving under the action of force field due to a (time-dependent) potential function \( \Phi(t,x) \), its de Broglie wave \( \psi(.,.) \) must satisfy the (relativistically invariant) partial differential equation (PDE),
\[ \left( \frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} \right) + \frac{4\pi i}{\hbar c^2} \Phi \frac{\partial \psi}{\partial t} - \frac{4\pi^2}{\hbar^2} \left( m_0 c^2 - \frac{\Phi^2}{c^2} \right) \psi_0 = 0. \tag{4} \]

This equation has important special cases.

First is the case where the potential function is time-independent. In this case we can take
\[ \psi(x,t) = a(x)e^{2\pi i (\nu' - \Phi(x))} = \psi_0(x)e^{2\pi i \nu'}. \]

A straightforward calculation then yields
\[
\frac{\partial^2 \psi_0}{\partial x^2} + \frac{4\pi^2 v^2}{c^2} \psi_0 - \frac{8\pi^2 v}{\hbar c^2} \Phi \cdot \psi_0 + \frac{4\pi^2}{\hbar^2} \left( m_0 c^2 - \frac{\Phi^2}{c^2} \right) \psi_0 = 0,
\]

or equivalently
\[
\frac{\partial^2 \psi_0}{\partial x^2} + \frac{4\pi^2 v^2}{c^2} \left( 1 - \frac{1}{\hbar v \Phi} \right) \psi_0 = 0
\]

where \( v_0 \) is the rest frequency: \( \hbar v_0 = m_0 c^2 \). Equation (5) like eq. (4) is relativistically invariant.

For slow particles for which \((\psi/c)^n\) can be neglected for \( n \geq 3 \), we have
\[
\hbar v = mc^2 = \frac{m_0}{\sqrt{1 - v^2/c^2}} c^2 = m_0 c^2 + \frac{1}{2} m_0 v^2
\]

where \( E \) is the total classical energy, given by \( E := (1/2) m_0 v^2 + \Phi \). For such particles \((E/\hbar v)^2\) and \((\Phi/\hbar v)^2\) can be neglected in eq. (5), which then, cf. (ref. 24), degenerates into the important time-independent (non-relativistic) Schrödinger equation for \( \psi_0 \) for slow particles, viz.
\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m_0}{\hbar^2} (E - \Phi) \psi = 0.
\]

The basic eq. (4) also yields Schrödinger’s (non-relativistic) time-dependent equation for slow particles. Taking
\[
\psi(t, x) = \psi_0(t, x) e^{i(2\pi \hbar) mc^2 x^2 / \hbar^2},
\]

we get from eq. (4), a PDE for \( \psi_0 \). When in this last PDE, we neglect small terms, noting that \( 1/\hbar \) is negligible compared to \( 1/c^2 \), since \( h \) is small, we get Schrödinger’s non-relativistic, parabolic equation for \( \psi_0 \):
\[
\frac{\hbar}{2\pi i} \frac{\partial \psi_0}{\partial t} = \frac{\hbar^2}{8\pi^2 m_0} \frac{\partial^2 \psi_0}{\partial x^2} + \Phi \cdot \psi_0,
\]

which is valid to a high degree of approximation for particles moving slowly compared to light. But for the imaginary coefficients, this is a diffusion equation.

For material systems of \( n \) degrees of freedom \((n > 1)\), it is difficult to get a relativistic invariant PDE for the de Broglie wave, from which the Schrödinger’s equation can be derived as an approximation. It was Schrödinger’s important work to directly get the following generalization of eq. (7)
\[
\frac{\hbar}{2\pi i} \frac{\partial \psi_0}{\partial t} + H(t, q, \frac{\hbar}{2\pi i} \frac{\partial}{\partial q}) \psi(q, t) = 0,
\]

where \( H \) is the Hamiltonian of the material system. Notice that this yields a wave in the configuration-spin space of the dynamical system. The solution \( \psi \) of eq. (8) is defined to be the de Broglie wave of the material system.

**B. Wave mechanical resonance and degeneracy**:
These phenomena, which are central to Haldane’s thought, stem from Bohr’s atomic theory. We know from this that an atom can be in one of several states 1,2,3, . . . , each with its energy level \( E_1, E_2, E_3, \ldots \), and that the atom transits from a lower state \( m \) to a higher state \( n (m < n) \) when it receives external radiant energy of frequency
\[
v_{n,m} = \frac{1}{\hbar} (E_n - E_m),
\]

and that when it transits from a higher state \( n \) to a lower state \( m \), it emits radiant energy of the same frequency \( v_{n,m} \).

Now suppose that we have two atoms \( a \) and \( b \), whose energy spectra have a common frequency \( v_0 \), say
\[
v_{m,m} = v_0 = v_{i,k}, \quad m < n \text{ and } i < k,
\]

so that by eq. (9),
\[
E_m^a - E_n^a = E_i^b - E_k^b.
\]

The atoms \( a \) and \( b \) are then said to be in resonance.

An energy exchange can take place between atoms \( a \) and \( b \) in resonance, even when they are loosely coupled. For suppose the atom \( a \) goes from state \( n \) to state \( m \) \((m < n)\), liberating radiant energy \( \hbar v_{m,n} = E_n^a - E_m^a > 0 \). This energy will be picked up by the atom \( b \), per the equation
\[
E_n^a - E_m^a = \hbar v_{m,n} = E_i^b - E_k^b.
\]

Now consider the single system \( c \), comprising both the atoms \( a \) and \( b \). The states of the system \( c \) will be integer pairs \((n, k)\) corresponding to the state \( n \) of \( a \) and the state \( k \) of \( b \). For the energy of \( c = \{a, b\} \) in this state we obviously have by eq. (12),
\[
E_{n,k}^c = E_n^a + E_k^b = E_m^a + E_i^b = E_{m,i}^a.
\]

The resonance of \( a \) and \( b \) thus introduces a degeneracy in the system \( c \): the states \((n, k)\) and \((m, i)\) of \( c \), which without resonance, would have been at different energy levels, are now at the same energy level. There is more than one mode of instant vibration associated with a given frequency.
Notice that when \(a, b\) are atoms of the same element (say neon gas), we have \(\nu_{m,n}^a = \nu_{m,n}^b\), for each \(m\) and \(n\). Hence there is ample scope for resonance and degeneracy unless \(a\) and \(b\) are very far apart. We see that for the biological molecules, which have plenty of repetitions of the same atoms, ranging from simple carbohydrates such as fats, to amino acids, and proteins, there will be a lot of resonance accompanied by degeneracy.

The subject of resonance is further discussed in Heisenberg\(^25\). The subject of the stability of biological molecules is treated in Schrödinger\(^26\).

C. The transparency of potential barriers: Tunnelling – Notable examples of overcoming a potential barrier by ‘tunnelling through its walls’ are offered by the phenomena of (i) the spontaneous emission of \(\alpha\)-particles (i.e. particles comprising 2 protons and 2 neutrons) by radioactive substances, and (ii) the absorption by lighter substances of outside \(\alpha\)-particles, and their transmutation to a different substance. Both phenomena were studied by E. Rutherford and F. Soddy during 1900–1910. What follows leans on S. Chandrasekhar\(^27\) and H. T. Flint\(^28\).

The situation before us is rather like that of a ball trapped inside a cylindrical hole of height \(h\) in a rough field (Figure 3). Clearly the ball can get out of the hole only if it receives a kinetic energy \(mv^2/2\), that exceeds \(mgh\). Now the nucleus of an atom may be conceived as a ‘hole’. The potential barrier and its strength \(V^*\) correspond to the ‘walls’ and to the quantity \(gh\), respectively, and the radius \(r^*\) of the nucleus corresponds to the radius of the hole.

For an \(\alpha\)-particle outside the nucleus of an atom, we have a strong repulsive Coulomb force between it and the nucleus, stemming from the positive charges that each carries. But for an \(\alpha\)-particle within the nucleus this repulsive force quickly diminishes, and gives way to a strong attractive force that prevents the \(\alpha\)-particle from escaping. Let \(V(r)\) be the potential energy (PE) of the \(\alpha\)-particle at distance \(r\) from the centre of the nucleus, positive values \(V(r)\) indicating repulsion, and negative values indicating attraction. The graph of \(V(r)\) is sketched in Figure 4.

We have
\[
V(r) = \frac{CC'}{r}, \quad \text{for } r \geq r^*,
\]
where \(C', C\) are the (positive) charges of the nucleus and of the \(\alpha\)-particle, respectively. It follows that the strength \(V^*\) of the potential barrier is given by
\[
V^* = V(r^*) = C'C/r^*
\]

For \(0 < r < r^*\), the graph is presumed to be a straightish curve of high slope, which extends below the \(x\)-axis.

According to classical theory an \(\alpha\)-particle inside the nucleus of the atom of uranium (like the ball in the hole) can get out only if its energy exceeds \(mV^*\), where \(m\) is the mass of the \(\alpha\)-particle. Rutherford found, however, that \(\alpha\)-particles emerging from the nuclei of the uranium atoms had much smaller kinetic energies than \(mV^*\), the energy needed to overcome the potential barrier. Reciprocally, it was found that when \(\alpha\)-particles are shot at lighter substances or even at uranium from outside with energies smaller than \(mV^*\), some penetrate into the nuclei. These phenomena, quite incomprehensible from the standpoint...
of classical physics, are explainable by wave mechanics as follows.

Since the equation of the graph of $V(r)$ on $[0, r^*]$ is not known, let us simplify matters a little by taking instead the PE.

$$V(r) = \begin{cases} \frac{1}{2} C - \frac{C}{r} & \text{for } r > r^* \\ V_0 & \text{for } 0 < r < r^* \end{cases}$$

The graph of this simpler $V(r)$ is shown in Figure 5.

The Schrödinger equation for the wave function $\psi(r)$ of the $\alpha$-particle of energy level $E$, with this simpler potential function $V(r)$ can be solved, and the probability density $|\psi(r)|^2$ of finding the $\alpha$-particle in the vicinity of $r$ can be calculated. It is found that even for $E < V^*$, $|\psi(r)|^2 > 0$ for $r^* < r < \infty$, and that $|\psi(r)|^2 \to 0$, exponentially, as $r \to \infty$. This explains the classically baffling, observational results of Rutherford and Soddy.

It will suffice to consider the case of a particle of rest mass $m_0$ and kinetic energy $E$ approaching a rectangular potential barrier of strength $V_0$ and with $w$ from the left (see Figure 6).

The potential function for this barrier is given by

$$\Phi(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ V_0 & \text{for } 0 < x \leq w \\ 0 & \text{for } w < x \end{cases}$$

These conditions entail that in region I (Figure 6) we have in addition to the incident wave a reflected wave of constant amplitude; in region II a highly damped wave travelling to the right, which is reflected at the barrier at $x = w$; in region III, a constant wave travelling to the right, but having a very small amplitude compared to the wave in region I. A fairly straightforward calculation yields the probability of finding the particle in region III:

$$|\psi_0(x)|^2 = \frac{E(V_0 - E)}{V_0^2} e^{-\frac{4\alpha\sqrt{V_0}}{\hbar}(2m_0(V_0 - E))}$$

for $w < x$.

This result is sufficiently indicative of other more complicated instances of tunnelling. It shows that the probability of tunnelling diminishes as the width $w$ of the barrier, and the ‘climbing’ $V_0 - E$ the particle has to do, increase.


**Notes**

1. Engels has further confused the issue by substituting ‘Nature’ for ‘matter’, thereby begging the most question ‘Is not Nature spiritual?’.
2. The Logos doctrine lends itself to scripture, and the higher religions identify the Logos with God, or perhaps with an essential attribute of God. Thus in the Bible we have ‘In the beginning, (i.e. in the foundation) was the Word, and the Word was with God, and the Word was God’, John, 1.1.

3. What prevailed earlier leaned on imprecise notions such as the ‘association of ideas’.

4. The deductive apparatus of Leibniz comprised the rules of logic together with his famous ‘Principle of Sufficient Reason’, which time does not allow us to describe. For an evaluation of the principle by a first-rate mathematician, see G. D. Birkhoff.

5. Note that these automata are machines in Ashby’s sense rather than in Wiener’s sense: after each communication (input–output cycle), the state of the automaton is changed. The communication is exclusively with God.

6. The omnipresence of the de Broglie wave of a particle, with respect to the frame of reference in which it is at rest, comes from setting $u = 0$ in the equation $V = c^2/u$ for the speed of the de Broglie wave, cf. Appendix, eq (3).

Received 4 October 1999; accepted 10 April 2000